

**Univerzitet u Nišu**  
**Fakultet zaštite na radu u Nišu**

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# **TEHNIČKA MEHANIKA**

## **REŠENI ZADACI SA VEŽBI**

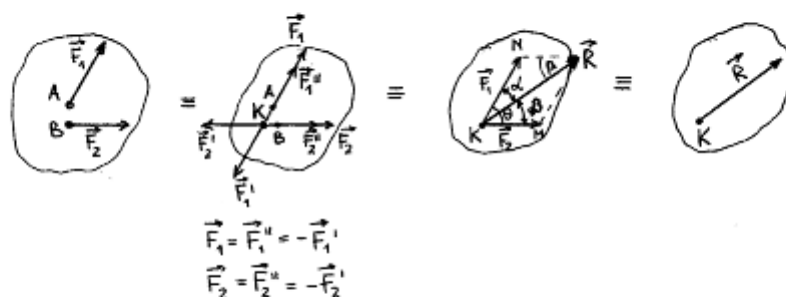
- Nerecenzirani materijal -

Niš, 2015.

# **STATIKA**

REZULTANTA KOMPLANARNIH SILA
ZADATAK BR. 1

Odrediti rezultantu komplanarnih sila intenziteta  $F_1 = 5$  [kN] i  $F_2 = 3$  [kN] koje napadaju kruto telo, tako da napadne linije sila grade među sobom ugao od  $60^\circ$ .



$$\begin{aligned} \vec{R} &= \vec{F}_1 + \vec{F}_2 \\ \vec{R} \cdot \vec{R} &= (\vec{F}_1 + \vec{F}_2) \cdot (\vec{F}_1 + \vec{F}_2) \\ R^2 &= F_1^2 + F_2^2 + 2F_1F_2 \cos \theta \\ R^2 &= 5^2 + 3^2 + 2 \cdot 5 \cdot 3 \cdot \cos 60^\circ \\ R^2 &= 49 \Rightarrow \underline{R = 7 \text{ [kN]}} \end{aligned}$$

$$\theta = \alpha + \beta$$

$$\frac{F_1}{\sin \beta} = \frac{F_2}{\sin \alpha} = \frac{R}{\sin \theta}$$

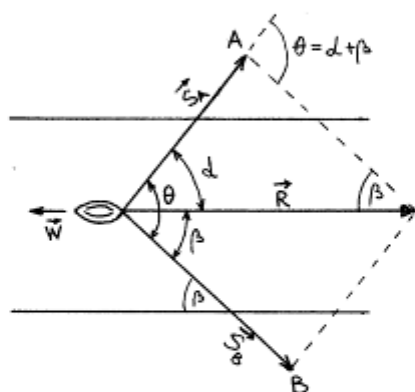
$$\sin \alpha = \frac{F_2}{R} \sin \theta = \frac{3}{7} \cdot \frac{\sqrt{2}}{2} \approx 0,3711 \Rightarrow \underline{\alpha \approx 22^\circ}$$

$$\beta = \theta - \alpha \Rightarrow \underline{\beta \approx 38^\circ}$$

**ZADATAK BR. 2**

Dva konja se kreću duž obala kanala konstantnom brzinom i pomoću dva užeta vuku čamac. Sile u užadima jednake su 80 [N], odnosno 96 [N], a ugao koji one zaklapaju jednak je 60°.

Naci otpor  $W$  vode kretanju čamca i uglove  $\alpha$  i  $\beta$  koje užad treba da zatvara sa obalama kanala da bi se čamac kretao paralelno obalama.



$$S_A = 80 \text{ [N]}$$

$$S_B = 96 \text{ [N]}$$

$$\theta = 60^\circ$$

$$\vec{W} = -\vec{R}$$

$$\vec{R} = \vec{S}_A + \vec{S}_B$$

$$\theta = \alpha + \beta$$

$$\frac{S_A}{\sin \beta} = \frac{S_B}{\sin \alpha} = \frac{R}{\sin \theta}$$

$$R^2 = S_A^2 + S_B^2 + 2 S_A S_B \cos \theta$$

$$R^2 = 80^2 + 96^2 + 2 \cdot 80 \cdot 96 \cdot \cos 60^\circ$$

$$R^2 = 23296$$

$$R = 16\sqrt{91} \text{ [N]} = 152,63 \text{ [N]}$$

$$\sin \beta = \frac{S_A}{R} \sin \theta = \frac{80}{16\sqrt{91}} \cdot \frac{\sqrt{3}}{2} \Rightarrow \beta \approx 27^\circ$$

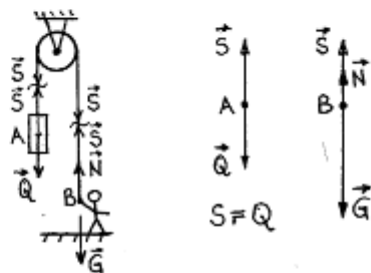
$$\alpha = \theta - \beta \Rightarrow$$

$$\alpha \approx 33^\circ$$

**ZADATAK BR. 3**

Na dnu okna nalazi se čovek težine  $G = 0,64 \text{ kN}$ .  
 Pomoću užeta, prebačenog preko nepokretnog kotura, čovek diže teret od  $0,48 \text{ kN}$ .

- a) Kolikom silom utiče čovek na dno okna?  
 b) Koliki najveći teret on može podići?



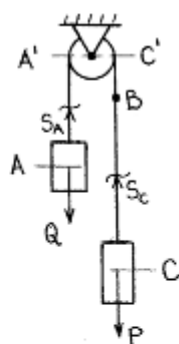
$$\begin{aligned}
 \text{a) } \vec{R} = 0 &\Rightarrow \vec{S} + \vec{N} + \vec{G} = 0 \\
 &S + N - G = 0 \\
 &N = G - S = G - Q \\
 &N = 0,64 - 0,48 \\
 &N = 0,16 \text{ [kN]}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } N = 0, S = G &\Rightarrow Q_{\max} = G \\
 &Q_{\max} = 0,64 \text{ [kN]}
 \end{aligned}$$

**ZADATAK BR. 4**

Teret  $Q = 30 \text{ kN}$  održava se u ravnoteži pomoću protivtega  $P$  koji je uvršćen. Uže, dužine  $10 \text{ m}$ , težine  $5 \text{ kN}$ , prebačeno je preko nepokretnog kotura  $B$ . Odrediti težinu  $P$  protivtega, sile  $S_A$ ,  $S_B$  i  $S_C$  na krajevima  $A$  i  $C$  užeta i silu na njegovoj sredini  $B$ , za slučaj:

- kada se tačke  $A$  i  $C$  nalaze na istom nivou;
- kada tačka  $A$  zauzima najviši položaj;
- kada tačka  $A$  zauzima najniži položaj.



$$Q = 30 \text{ [kN]}$$

$$l = 10 \text{ [m]}$$

$$Q_{uz} = 5 \text{ [kN]}$$

$$\overline{AB} = \overline{BC}$$

$$\text{a) } \overline{AA'} = \overline{CC'}$$

$$Q = P = 30 \text{ [kN]}$$

$$S_A = S_C = 30 \text{ [kN]}$$

$$S_B = Q + \frac{Q_{uz}}{2}$$

$$S_B = 32,5 \text{ [kN]}$$

$$\text{b) } A \equiv A'$$

$$Q = Q_{uz} + P$$

$$P = Q - Q_{uz}$$

$$P = 25 \text{ [kN]}$$

$$S_A = Q_{uz} + P$$

$$S_A = 30 \text{ [kN]}$$

$$S_B = \frac{Q_{uz}}{2} + P = 27,5 \text{ [kN]}$$

$$S_C = P = 25 \text{ [kN]}$$

$$\text{c) } C \equiv C'$$

$$Q + Q_{uz} = P$$

$$P = 35 \text{ [kN]}$$

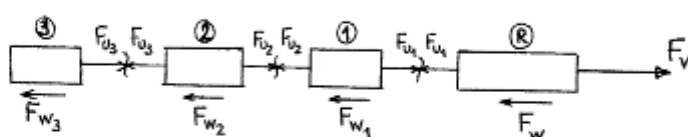
$$S_A = Q = 30 \text{ [kN]}$$

$$S_B = 32,5 \text{ [kN]}$$

$$S_C = P = 35 \text{ [kN]}$$

**ZADATAK BR. 5**

Remorker vuče tri šlepa različitih veličina. Šlepi su povezani užadima tako da slede jedan za drugim. U datom trenutku, vučna sila remorkera jednaka je 1800 [N]. Otpor vode kretanju remorkera jednak je 600 [N], prvog šlepa 600 [N], drugog 400 [N], a trećeg 200 [N]. Na raspolaganju stoji uža koje sa sigurnošću može da prenosi silu od 200 [N]. Sa koliko užadi treba spojiti prvi šlep sa remorkerom, drugi šlep sa prvim i treći sa drugim?



$$F_v = 1800 \text{ [N]}; F_w = 600 \text{ [N]}; F_{w1} = 600 \text{ [N]}; F_{w2} = 400 \text{ [N]}; F_{w3} = 200 \text{ [N]}; F_u = 200 \text{ [N]}$$

$$R: F_v = F_w; F_{u1} = 0$$

$$1: F_{u1} = F_v - F_w = 1800 - 600 = 1200 \text{ [N]}$$

$$n_1 = \frac{F_{u1}}{F_u} = \frac{1200}{200} = \underline{\underline{6 \text{ užadi}}}$$

$$2: F_{u1} - F_{w1} - F_{u2} = 0$$

$$F_{u2} = F_{u1} - F_{w1} = 1200 - 600 = 600 \text{ [N]}$$

$$n_2 = \frac{F_{u2}}{F_u} = \frac{600}{200} = \underline{\underline{3 \text{ užadi}}}$$

$$3: F_{u2} - F_{w2} - F_{u3} = 0$$

$$F_{u3} = F_{u2} - F_{w2} = 600 - 400 = 200 \text{ [N]}$$

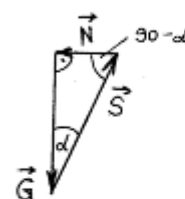
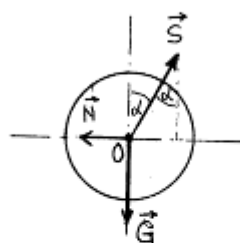
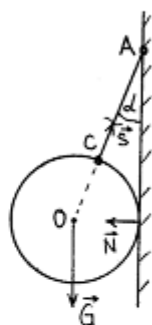
$$n_3 = \frac{F_{u3}}{F_u} = \frac{200}{200} = \underline{\underline{1 \text{ uže}}}$$

$$F_{u3} - F_{w3} = 0$$

$$F_{u3} = F_{w3} = 200 \text{ [N]}$$

SISTEM KOMPLANARNIH SUČELJNIH SILA
ZADATAK BR. 6

O vertikalni glatki zid  $\overline{AB}$  oslonjena je kugla  $O$ , obešena o konac  $AC$ . Ugao koji konac zatvara sa zidom je  $\alpha$ , a težina kugle je  $G$ . Odrediti silu  $S$  u koncu i pritisak  $N$  kugle na zid.



$$\frac{S}{\sin 90^\circ} = \frac{N}{\sin \alpha} = \frac{G}{\sin (90^\circ - \alpha)}$$

$$S = G \frac{\sin 90^\circ}{\cos \alpha} = \frac{G}{\cos \alpha}$$

$$N = G \frac{\sin \alpha}{\cos \alpha} = G \operatorname{tg} \alpha$$

II način:

$$\vec{R} = 0$$

$$\vec{R} = R_x \vec{i} + R_y \vec{j}$$

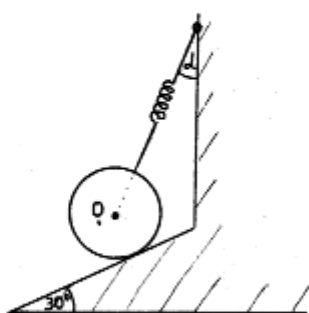
$$R_x = \sum X_i = 0 \Rightarrow S \cos (90^\circ - \alpha) - N = 0 \Rightarrow S = \frac{N}{\sin \alpha}$$

$$R_y = \sum Y_i = 0 \Rightarrow S \sin (90^\circ - \alpha) - G = 0 \Rightarrow S = \frac{G}{\cos \alpha} \Rightarrow N = G \operatorname{tg} \alpha$$

**ZADATAK BR. 7**

Pomoću konca održava se na glatkoj strmoj ravni kugla O, težine 20 [kN]. Konac je vezan za opružnu vagu, koja pokazuje silu od 10 [kN]. Strma ravan zatvara sa horizontalalom ugao od 30°.

Odrediti ugao  $\alpha$  koji konac zatvara sa vertikalom i pritisak N kugle na strmu ravan.



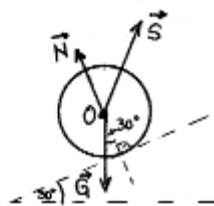
$$G = 20 \text{ [kN]}$$

$$S = 10 \text{ [kN]}$$

$$\varphi = 30^\circ$$

$$\alpha = ?$$

$$N = ?$$



$$\frac{S}{\sin 30^\circ} = \frac{N}{\sin \alpha} = \frac{G}{\sin (180^\circ - 30^\circ - \alpha)}$$

$$\sin (150^\circ - \alpha) = \frac{G}{S} \sin 30^\circ$$

$$\sin 150^\circ \cos \alpha - \cos 150^\circ \sin \alpha = \frac{G}{S} \sin 30^\circ$$

$$\frac{1}{2} \cos \alpha + \frac{\sqrt{3}}{2} \sin \alpha = \frac{20}{10} \cdot \frac{1}{2}$$

$$\cos \alpha + \sqrt{3} \sin \alpha = 2$$

$$-\cos \alpha + 2 = \sqrt{3} \sin \alpha / 2$$

$$\cos^2 \alpha + 4 - 4 \cos \alpha = 3 \sin^2 \alpha ; \quad \sin^2 \alpha = 1 - \cos^2 \alpha$$

$$\cos^2 \alpha + 4 - 4 \cos \alpha = 3 - 3 \cos^2 \alpha$$

$$4 \cos^2 \alpha - 4 \cos \alpha + 1 = 0$$

$$\cos \alpha_{1,2} = \frac{4 \pm \sqrt{16 - 4 \cdot 4 \cdot 1}}{8} = \frac{1}{2}$$

$$\cos \alpha = \frac{1}{2} \Rightarrow \underline{\alpha = 60^\circ}$$

$$N = S \frac{\sin 60^\circ}{\sin 30^\circ} \Rightarrow \underline{N = 10\sqrt{3} \text{ [kN]}}$$

**ZADATAK BR. 8**

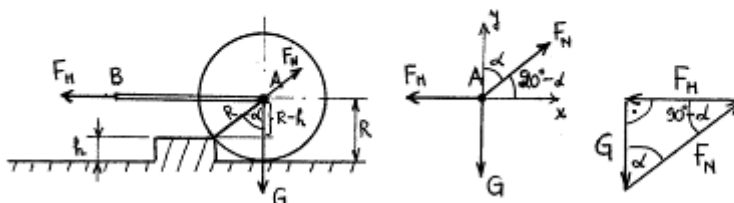
Kojom horizontalnom silom  $F_H$  treba vući držalju valjka da bi prešao prag visine  $h = 8$  [cm]. Poluprečnik valjka  $R = 60$  [cm], a njegova težina  $G = 2$  [kN].

$$F_H = ?$$

$$G = 2 \text{ [kN]}$$

$$h = 8 \text{ [cm]}$$

$$R = 60 \text{ [cm]}$$



$$\frac{F_H}{\sin d} = \frac{F_N}{\sin 90^\circ} = \frac{G}{\sin(90^\circ - d)}$$

$$F_H = G \frac{\sin d}{\cos d} = G \operatorname{tg} d$$

$$\cos d = \frac{R-h}{R}$$

$$\operatorname{tg} d = \frac{\sin d}{\cos d} = \frac{\sin d}{\frac{R-h}{R}} = \frac{R \sin d}{R-h} = \frac{R \sqrt{1 - \left(\frac{R-h}{R}\right)^2}}{R-h} = \frac{\sqrt{R^2 - R^2 + 2Rh + h^2}}{R-h} = \frac{\sqrt{2Rh + h^2}}{R-h}$$

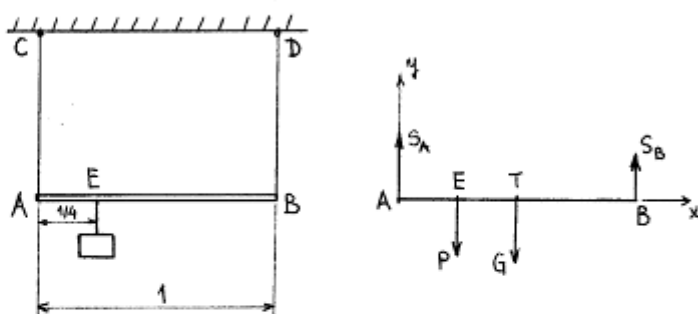
$$\operatorname{tg} d = \frac{\sqrt{2 \cdot 60 \cdot 8 + 8^2}}{60 - 8} = \frac{\sqrt{896}}{52} = 0,5756 \Rightarrow d = 29,926^\circ$$

$$F_H = G \operatorname{tg} d = 2 \cdot 0,5756 \Rightarrow F_H = 1,1522 \text{ [kN]}$$

**SISTEM KOMPLANARNIH PARALELNIH SILA**

**ZADATAK BR. 9**

Homogeni štap AB, dužine 1[m], težine 2[kN], obešen je horizontalno o dva kanapa AC i BD. Za štap je obešen u tački E teret P=12[kN]. Odstojanje AE =  $\frac{1}{4}$  [m]. Odrediti sile u kanapima.



$$1) \sum X_i = 0$$

$$2) \sum Y_i = S_A - P - G + S_B = 0$$

$$3) \sum \overset{\curvearrowright}{M}_A = -P \cdot \frac{1}{4} - G \cdot \frac{1}{2} + S_B \cdot 1 = 0$$

$$S_B = \frac{P}{4} + \frac{G}{2}$$

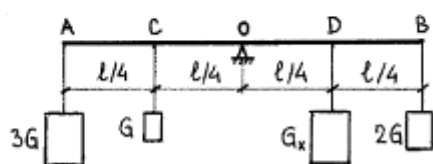
$$S_B = 3 + 1 \rightarrow \underline{S_B = 4 \text{ [kN]}}$$

$$2) \rightarrow S_A = P + G - S_B$$

$$S_A = 12 + 2 - 4 \rightarrow \underline{S_A = 10 \text{ [kN]}}$$

**ZADATAK BR. 10**

O tačke A i B ravnokrake dvokrake poluge AB obešeni su tereti težina  $3G$  i  $2G$ , a o tačku C ( $\overline{AC} = \overline{CO}$ ) teret težine  $G$ . Kolikim se teretom mora delovati u tački D ( $\overline{OD} = \overline{DB}$ ) da bi poluga bila u ravnoteži?



$$\overline{AO} = \overline{OB} ; \overline{AC} = \overline{CO} ; \overline{OD} = \overline{DB}$$

$$\sum M_O = 0$$

$$3G \frac{l}{2} + G \frac{l}{4} = 2G \frac{l}{2} + G_x \cdot \frac{l}{4} \quad / \cdot \frac{4}{l}$$

$$\underline{G_x = 4G \left( \frac{3}{2} + \frac{1}{4} - 1 \right) = 3G}$$

**ZADATAK BR. 11**

Duž horizontalne grede na dva oslonca mogu se pomerati dva tereta, jedan  $C = 200$  [kN], drugi  $D = 100$  [kN]. Raspon grede je  $L = 4$  [m],  $\overline{CD} = 1$  [m]. Na kom rastojanju  $x$  od oslonca A mora da se nalazi teret C da bi otpor oslonca A bio dva puta veći od otpora oslonca B? Uticaj težine grede zanemariti.

$$C = 200 \text{ [kN]}$$

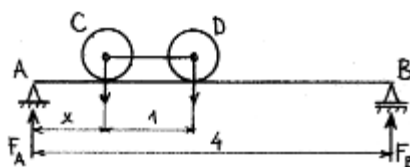
$$D = 100 \text{ [kN]}$$

$$\overline{AB} = 4 \text{ [m]}$$

$$\overline{CD} = 1 \text{ [m]}$$

$$x = ?$$

$$F_A = 2F_B$$



$$\sum Y_i = F_A + F_B - C - D = 0$$

$$F_A = 2F_B \rightarrow 3F_B = C + D, \quad F_B = \frac{C + D}{3}$$

$$\underline{F_B = 100 \text{ [kN]} ; \quad F_A = 200 \text{ [kN]}}$$

$$\sum \overset{\curvearrowright}{M}_A = F_B \cdot \overline{AB} - D(x + \overline{CD}) - Cx = 0$$

$$x = \frac{F_B \cdot \overline{AB} - D \cdot \overline{CD}}{C + D} = \frac{100 \cdot 4 - 100 \cdot 1}{200 + 100} = \frac{300}{300}$$

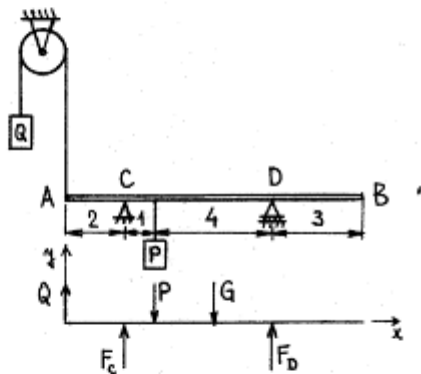
$$\underline{x = 1 \text{ [m]}}$$

**ZADATAK BR. 12**

Homogena prizmatična greda AB, dužine 10 m, težine 200 kN, oslonjena je u tačkama C i D. Rastojanje oslonca C od kraja A grede jednako je 2 m, rastojanje oslonca D od kraja B grede jednako je 3 m. Na rastojanju 3 m od kraja A grede obešen je teret  $P = 800$  kN. Za kraj A grede vezano je uže koje je prebačeno preko kotura i zategnuto teretom  $Q = 300$  kN. Odrediti otpore oslonaca.

$$\begin{aligned}
 \overline{AB} &= 10 \text{ [m]} & P &= 800 \text{ [kN]} \\
 \overline{AC} &= 2 \text{ [m]} & Q &= 300 \text{ [kN]} \\
 \overline{DB} &= 3 \text{ [m]} & G &= 200 \text{ [kN]} \\
 \overline{AP} &= 3 \text{ [m]} & &
 \end{aligned}$$

$$F_C, F_D = ?$$



$$1) \sum Y_i = 0 : Q + F_C - P - G + F_D = 0$$

$$2) \sum M_A = 0 : F_C \cdot 2 - P \cdot 3 - G \cdot 5 + F_D \cdot 7 = 0$$

$$\begin{aligned}
 1) \Rightarrow F_C + F_D &= P + G - Q & \Rightarrow F_C + F_D &= 700 \quad / \cdot (-2) \\
 2) \Rightarrow 2F_C + 7F_D &= 3P + 5G & \Rightarrow 2F_C + 7F_D &= 3400
 \end{aligned}
 \left. \vphantom{\begin{aligned} 1) \Rightarrow F_C + F_D &= P + G - Q \\ 2) \Rightarrow 2F_C + 7F_D &= 3P + 5G \end{aligned}} \right\} + \Rightarrow 5F_D = 2000$$

$$F_D = 400 \text{ [kN]}$$

$$F_C = 300 \text{ [kN]}$$

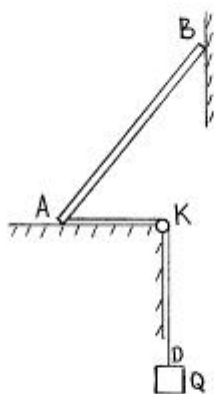
**SISTEM PROIZVOLJNIH KOMPLANARNIH SILA**
**ZADATAK BR. 13**

Homogeni štap AB, dužine  $l$ , težine  $G$ , oslanja se na vertikalni zid krajem B i na pod krajem A. Za kraj A vezano je lako uže, koje je prebačeno preko malog kotura K, o čiji drugi kraj D visi teret  $Q$ .

Odrediti ugao koji štap AB gradi sa horizontalnim podom ako je  $G/Q = 2\sqrt{3}$ .

Odrediti reakcije u tačkama oslanjanja ako je  $Q = 20$  [kN].  
Trenje zanemariti.

$$Q = 20 \text{ [kN]}, \quad G = Q \cdot 2\sqrt{3} = 40\sqrt{3} \text{ [kN]}$$



$$\sum X_i = 0: Q - F_B = 0 \Rightarrow F_B = Q = 20 \text{ [kN]}$$

$$\sum Y_i = 0: F_A - G = 0 \Rightarrow F_A = G = 40\sqrt{3} \text{ [kN]}$$

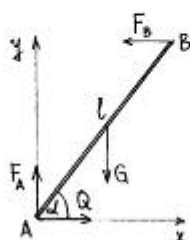
$$\sum M_A = 0: F_B \cdot l \sin \alpha - G \cdot \frac{l}{2} \cos \alpha = 0$$

$$F_B \sin \alpha = \frac{1}{2} G \cos \alpha$$

$$\frac{\sin \alpha}{\cos \alpha} = \frac{G}{2F_B}$$

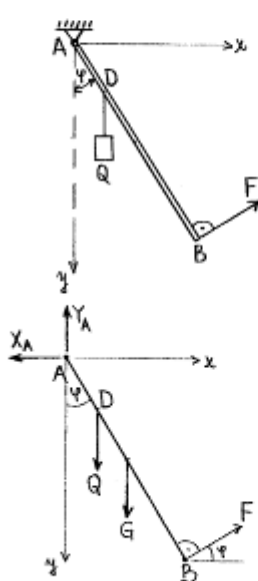
$$\operatorname{tg} \alpha = \frac{40\sqrt{3}}{2 \cdot 20} = \sqrt{3}$$

$$\alpha = \arctg \sqrt{3} = 60^\circ$$



**ZADATAK BR. 14**

Homogeni štap AB, dužine  $L$  i težine  $G$ , zglobno je vezan u tački A. U tački D ( $\overline{AD} = L/4$ ) obešen je teret  $Q$ , a na kraj B štapa dejstvuje koncentrisana sila  $F$  upravno na osu štapa. Za ravnotežni položaj štapa odrediti ugao koji osa štapa gradi sa vertikalom i silu u zglobovima, ako je  $G = 50 \text{ kN}$ ,  $Q = 4F = 3G$ ,  $L = 0,8 \text{ m}$ .



$$\Sigma M_A = 0: F \cdot \frac{L}{2} - G \frac{L}{2} \sin \varphi - Q \frac{L}{4} \sin \varphi = 0 \quad / \cdot 4$$

$$4F = (2G + Q) \sin \varphi$$

$$\sin \varphi = \frac{4F}{2G + Q} = \frac{3G}{2G + 3G} = \frac{3}{5}$$

$$\cos \varphi = \sqrt{1 - \sin^2 \varphi} = \frac{4}{5}$$

$$\Sigma X_i = 0: X_A - F \cos \varphi = 0 \Rightarrow X_A = F \cos \varphi = \frac{4}{5} F = \frac{3}{5} G$$

$$X_A = 30 \text{ [kN]}$$

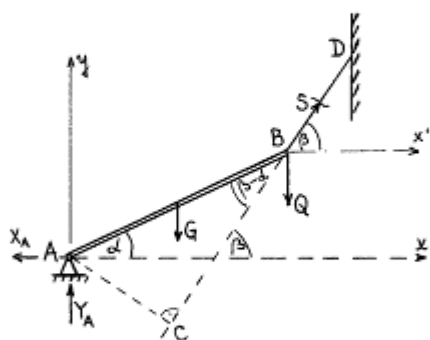
$$\Sigma Y_i = 0: -Y_A + Q + G - F \sin \varphi = 0$$

$$Y_A = Q + G - F \sin \varphi = 3G + G - \frac{3}{5} G = \frac{71}{20} G$$

$$Y_A = 177,5 \text{ [kN]}$$

**ZADATAK BR. 15**

Homogeni štap  $\overline{AB}$ , dužine  $L$ , težine  $G$ , vezan je krajem  $A$  zglibom za pod. Drugi njegov kraj  $B$  vezan je užetom za zid. O kraj  $B$  štapa visi teret  $Q$ . Odrediti sile veza ako štap gradi ugao  $\alpha$  sa horizontalom, a uže ugao  $\beta$ .



$$1) \sum X_i = 0: -X_A + S \cos \beta = 0 \Rightarrow X_A = S \cos \beta$$

$$2) \sum Y_i = 0: Y_A - G - Q + S \sin \beta = 0 \Rightarrow Y_A = G + Q - S \sin \beta$$

$$3) \sum M_A = 0: -G \frac{L}{2} \cos \alpha - Q \cdot L \cos \alpha + S \cdot L \sin(\beta - \alpha) = 0$$

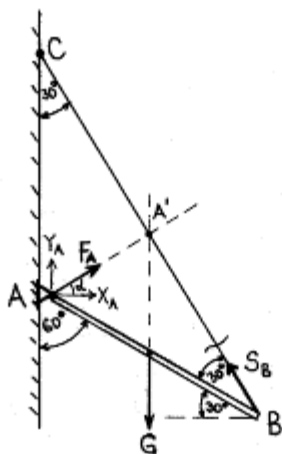
$$S = \frac{(G + 2Q) \cos \alpha}{2 \sin(\beta - \alpha)}$$

$$1) \Rightarrow X_A = \frac{(G + 2Q) \cos \alpha}{2 \sin(\beta - \alpha)} \cdot \cos \beta$$

$$2) \Rightarrow Y_A = G + Q - \frac{(G + 2Q) \cos \alpha}{2 \sin(\beta - \alpha)} \sin \beta$$

**ZADATAK BR. 16**

Homogeni prizmatični štap  $\overline{AB}$  vezan je krajem A zglavkasto za zid, a kraj B pridržava uže BC. Osa štapa zatvara sa vertikalom ugao od  $60^\circ$ . Ugao  $ABC = 30^\circ$ . Težina štapa iznosi  $G = 2$  [kN]. Odrediti brojnu vrednost, pravac i smer otpora A i silu u užetu  $S_B$ .



$$1) \sum X_i = 0: X_A - S_B \cos 60^\circ = 0$$

$$2) \sum Y_i = 0: Y_A - G + S_B \sin 60^\circ = 0$$

$$3) \sum M_A = 0: S_B \cdot \overline{AA'} - G \frac{\overline{AB}}{2} \sin 60^\circ = 0$$

$$S_B = \frac{G}{2} \frac{\overline{AB}}{\overline{AA'}} \sin 60^\circ$$

$$\overline{AA'} = \overline{AB} \cdot \sin 30^\circ$$

$$S_B = \frac{G}{2} \frac{\overline{AB}}{\overline{AB} \sin 30^\circ} \sin 60^\circ = \frac{G}{2} \frac{\sqrt{3}}{\frac{1}{2}}$$

$$S_B = G \frac{\sqrt{3}}{2} = \sqrt{3} \text{ [kN]}$$

$$1) \Rightarrow X_A = S_B \cos 60^\circ = G \frac{\sqrt{3}}{2} \cdot \frac{1}{2} \Rightarrow X_A = G \frac{\sqrt{3}}{4}$$

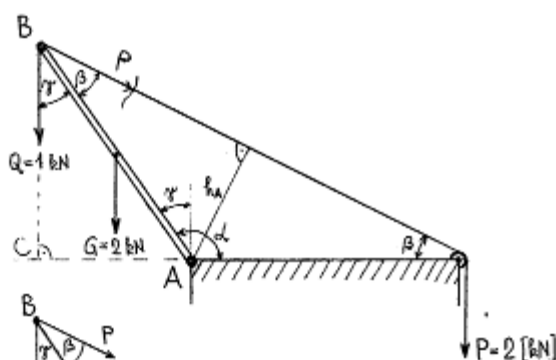
$$2) \Rightarrow Y_A = G - S_B \sin 60^\circ = G - G \frac{\sqrt{3}}{2} \frac{\sqrt{3}}{2} \Rightarrow Y_A = \frac{G}{4}$$

$$F_A = \sqrt{X_A^2 + Y_A^2} = \sqrt{\left(\frac{G\sqrt{3}}{4}\right)^2 + \left(\frac{G}{4}\right)^2} = \frac{G}{4} \sqrt{3+1} \Rightarrow F_A = \frac{1}{2} G = 1 \text{ [kN]}$$

$$\operatorname{tg} \alpha = \frac{Y_A}{X_A} = \frac{\frac{G}{4}}{\frac{G\sqrt{3}}{4}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \Rightarrow \alpha = \operatorname{arctg} \left( \frac{\sqrt{3}}{3} \right), \alpha = 30^\circ$$

**ZADATAK BR. 17**

O štap AB, koji se može obrtati oko zgloba A, obešen je u tački B, pomoću konca, teg G, težine 1 kN. Za kraj B štapa AB vezan je drugi konac koji je prebačen preko nepokretnog kotura D i zategnut tegom od 2 kN. Dužina  $\overline{AB} = \overline{AD}$ . Težina štapa jednaka je 2 kN. Odrediti veličinu ugla  $\sphericalangle DAB = \alpha$  pri kojem će štap AB stajati u ravnoteži. Trenje zanemariti.



$$\begin{aligned}
 \sum \vec{M}_A = 0 : & P \cdot \overline{AB} \sin \beta - Q \cdot \overline{AB} \sin \gamma - G \frac{\overline{AB}}{2} \sin \gamma = 0 \\
 P \sin \beta &= \left( Q + \frac{G}{2} \right) \sin \gamma \\
 2 \sin \beta &= \left( 1 + \frac{2}{2} \right) \sin \gamma \\
 \sin \beta &= \sin \gamma \\
 \beta &= \gamma
 \end{aligned}$$

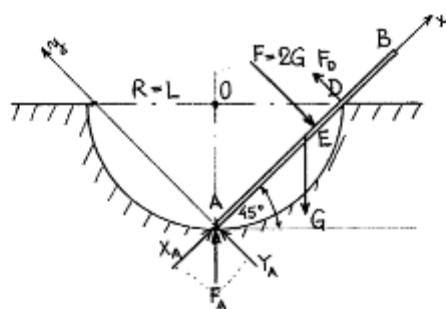
$$\Delta ABC : \sphericalangle CAB = 90 - \gamma$$

$$\left. \begin{aligned}
 90 - \gamma &= 180 - \alpha \\
 \alpha + 2\beta &= 180
 \end{aligned} \right\} \Rightarrow \left. \begin{aligned}
 \alpha - \gamma &= 90 \quad / \cdot (-1) \\
 \alpha + 2\gamma &= 180
 \end{aligned} \right\} + \Rightarrow 3\gamma = 90^\circ \Rightarrow \underline{\gamma = \beta = 30^\circ}$$

$$\Rightarrow \underline{\alpha = 120^\circ}$$

**ZADATAK BR. 18**

Homogeni štap  $\overline{AB}$ , dužine  $2L$ , težine  $G$ , postavljen je u unutrašnjost glatkog polukružnog prstena, poluprečnika  $R=L$ , u vertikalnoj ravni. Na kom rastojanju  $\overline{AE}$  treba dejstvovati silom  $F=2G$ , upravno na štap, da bi zauzeo položaj ravnoteže tako da se kraj  $A$  štapa nalazi na istoj vertikali sa centrom polukružnog prstena? Odrediti reakcije u tačkama oslanjanja štapa.



$$\begin{aligned} \overline{AB} &= 2L ; G \\ R &= L \\ F &= 2G \end{aligned}$$

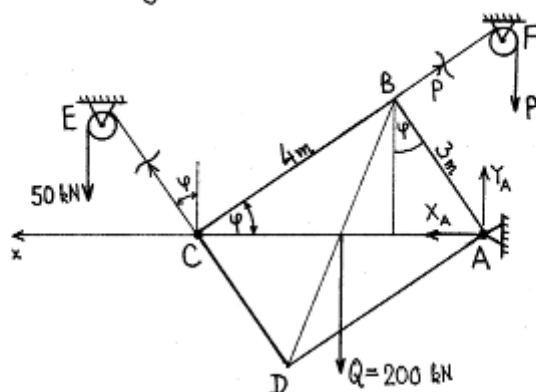
$$\begin{aligned} 1) \sum X_i = 0 : X_A = G \sin 45^\circ &\rightarrow X_A = G \frac{\sqrt{2}}{2} \\ X_A = F_A \sin 45^\circ = F_A \frac{\sqrt{2}}{2} &\left. \vphantom{X_A = G \frac{\sqrt{2}}{2}} \right\} \rightarrow F_A = G \end{aligned}$$

$$\begin{aligned} 2) \sum Y_i = 0 : Y_A - G \cos 45^\circ - 2G + F_D &= 0 \\ G \frac{\sqrt{2}}{2} - G \frac{\sqrt{2}}{2} - 2G + F_D &= 0 \Rightarrow F_D = 2G \end{aligned}$$

$$\begin{aligned} 3) \sum M_A = 0 : G \cdot L \cos 45^\circ + 2G \cdot \overline{AE} - F_D \cdot L\sqrt{2} &= 0 \\ G \cdot L \frac{\sqrt{2}}{2} + 2G \cdot \overline{AE} - 2G L \sqrt{2} &= 0 \\ \sqrt{2} L + 4 \overline{AE} - 4 \sqrt{2} L &= 0 \\ 4 \overline{AE} = 3 \sqrt{2} L \\ \overline{AE} = \frac{3\sqrt{2}}{4} L \approx 1,056 L \end{aligned}$$

**ZADATAK BR. 19**

Homogena pravougaona ploča ABCD, težine  $Q$ , može se obrtati u vertikalnoj ravni oko nepomičnog zgloba A. Ploča se održava u datom ravnotežnom položaju pomoću užadi koja su prebačena preko glatkih nepokretnih koturova E i F i zategnuta silama. Odrediti intenzitet sile  $P$  i komponente otpora zgloba A, kada je  $Q = 200 \text{ kN}$ , levo uže zategnuto silom od  $50 \text{ [kN]}$  i  $\cos \varphi = \frac{4}{5}$ .



$$\cos \varphi = \frac{4}{5} \Rightarrow \sin \varphi = \frac{3}{5}$$

$$\overline{AC} = d = 5 \text{ m}$$

- 1)  $\sum X_i = 0 : 50 \cdot \sin \varphi - P \cos \varphi + X_A = 0$
- 2)  $\sum Y_i = 0 : 50 \cos \varphi + P \sin \varphi - Q + Y_A = 0$
- 3)  $\sum \vec{M}_A = 0 : 50 \cdot 4 + P \cdot 3 - 200 \cdot 2,5 = 0 \Rightarrow \underline{P = 100 \text{ [kN]}}$

$$1) \rightarrow X_A = -50 \cdot \frac{3}{5} + 100 \cdot \frac{4}{5} ; \underline{X_A = 50 \text{ [kN]}}$$

$$2) \rightarrow Y_A = 200 - 50 \cdot \frac{4}{5} - 100 \cdot \frac{3}{5} ; \underline{Y_A = 100 \text{ [kN]}}$$

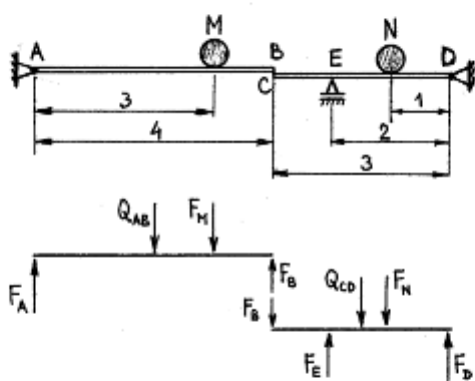
$$F_A = \sqrt{X_A^2 + Y_A^2} = \sqrt{50^2 + 100^2} = 50 \sqrt{1 + 2^2} = 50 \sqrt{5} \text{ [kN]}$$

$$\operatorname{tg} \alpha = \frac{Y_A}{X_A} = \frac{100}{50} = 2$$

$$\underline{\alpha = \operatorname{arctg} 2 = 63^\circ 30'} ; \underline{F_A \approx 112 \text{ [kN]}}$$

**ZADATAK BR. 20**

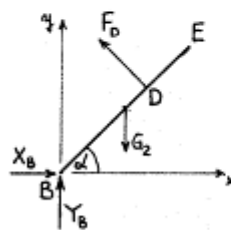
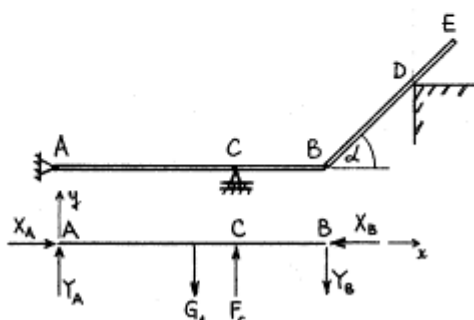
Homogena prizmatična greda AB, dužine 4 m, težine 20 kN, vezana je u tački A zglavkasto za zid, a krajem B oslanja se na drugu homogenu prizmatičnu gredu CD, dužine 3 m, težine 16 kN. Greda CD je poduprta u tački E, a u tački D je zglavkasto vezana za zid. U tačkama M i N dejstvuju tereti, svaki po 8 kN. Odrediti otpore oslonaca.



$$\begin{aligned} \overline{AB}: & \quad 1) \sum Y_i = 0 : F_A - Q_{AB} - F_M + F_B = 0 \\ & \quad 2) \sum \overset{\circlearrowleft}{M}_B = 0 : F_A \cdot 4 - Q_{AB} \cdot 2 - F_M \cdot 1 = 0 \Rightarrow F_A = \frac{2Q_{AB} + F_M}{4} = \frac{2 \cdot 20 + 8}{4} ; \underline{F_A = 12 \text{ [kN]}} \\ & \quad 1) \Rightarrow F_B = Q_{AB} + F_M - F_A = 20 + 8 - 12 \Rightarrow \underline{F_B = 16 \text{ [kN]}} \\ \overline{BD}: & \quad 3) \sum Y_i = 0 : -F_B + F_E - Q_{CD} - F_N + F_D = 0 \\ & \quad 4) \sum \overset{\circlearrowleft}{M}_D = 0 : -F_B \cdot 3 + F_E \cdot 2 - Q_{CD} \cdot 1.5 - F_N \cdot 1 = 0 \\ & \quad \quad \quad F_E = \frac{1}{2} (3F_B + Q_{CD} \cdot 1.5 + F_N) = \frac{1}{2} (3 \cdot 16 + 16 \cdot 1.5 + 8) \Rightarrow \underline{F_E = 40 \text{ [kN]}} \\ & \quad 3) \Rightarrow F_D = F_B + Q_{CD} + F_N - F_E = 16 + 16 + 8 - 40 \Rightarrow \underline{F_D = 0} \end{aligned}$$

**ZADATAK BR. 21**

Horizontalna greda  $\overline{AB}$ , težine  $G_1 = 20 \text{ kN}$ , pričvršćena je za zid zglobovom A i oslanja se na oslonac C. Za kraj B grede zglobovom je pričvršćena greda  $\overline{BE}$ , težine  $G_2 = 40 \text{ kN}$ , koja se oslanja na ispust D. Pri tome je:  $\overline{CB} = \frac{1}{3}\overline{AB}$ ,  $\overline{DE} = \frac{1}{3}\overline{BE}$  i  $\alpha = 45^\circ$ .  
Odrediti reakcije oslonaca.



GREDA  $\overline{BE}$ :

$$\begin{aligned}
 1) \sum X_i = 0: & X_B - F_D \sin \alpha = 0 \\
 & X_B = F_D \sin \alpha \\
 2) \sum Y_i = 0: & Y_B - G_2 + F_D \cos \alpha = 0 \\
 & Y_B = G_2 - F_D \cos \alpha \\
 3) \sum M_B = 0: & F_D \cdot \frac{2}{3} \overline{BE} - G_2 \cdot \frac{1}{2} \overline{BE} \cos \alpha = 0 \\
 & F_D = \frac{3}{2} G_2 \cdot \frac{1}{2} \cos \alpha \\
 & F_D = \frac{3}{4} \cdot 40 \cdot \frac{\sqrt{2}}{2} \\
 & F_D = 21,2 \text{ [kN]} \\
 1) \Rightarrow & X_B = 15 \text{ [kN]} \\
 2) \Rightarrow & Y_B = 25 \text{ [kN]} \\
 F_B = & \sqrt{X_B^2 + Y_B^2} = 29,15 \text{ [kN]}
 \end{aligned}$$

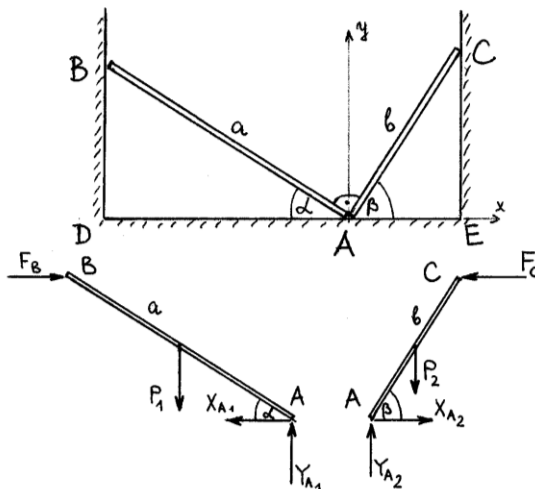
GREDA  $\overline{AB}$ :

$$\begin{aligned}
 1) \sum X_i = 0: & X_A - X_B = 0 \Rightarrow X_A = X_B = 15 \text{ [kN]} \\
 2) \sum Y_i = 0: & Y_A - G_1 + F_C - Y_B = 0 \\
 3) \sum M_A = 0: & -Y_B \cdot \overline{AB} + F_C \cdot \frac{2}{3} \overline{AB} - G_1 \cdot \frac{1}{2} \overline{AB} = 0 \\
 & F_C = \frac{3}{2} \left( \frac{1}{2} G_1 + Y_B \right) = \frac{3}{2} \left( \frac{1}{2} \cdot 20 + 25 \right) \\
 & F_C = 52,5 \text{ [kN]} \\
 2) \Rightarrow & Y_A = G_1 - F_C + Y_B = 20 - 52,5 + 25 \\
 & Y_A = -7,5 \text{ [kN]} \\
 F_A = & \sqrt{X_A^2 + Y_A^2} = 16,77 \text{ [kN]}
 \end{aligned}$$

## ZADATAK BR. 22

Dva homogena štapa  $\overline{AB}$  i  $\overline{AC}$  oslanjaju se u tački A na glatki horizontalni zid i naslanjaju se jedan na drugi. U tački B, odnosno C, štapovi se oslanjaju na vertikalne zidove.

Odrediti razmak  $\overline{DE}$  između zidova pri kojem ose štapova, u ravnotežnom položaju sistema, zaklapaju među sobom ugao od  $90^\circ$ . Dužina štapa  $\overline{AB} = a$ ,  $\overline{AC} = b$ . Težina štapa  $\overline{AB}$  jednaka je  $P_1$ , a štapa  $\overline{AC}$  jednaka je  $P_2$ .



$$\overline{DE} = a \cos \alpha + b \cos \beta$$

$$\alpha + \beta = 90^\circ \Rightarrow \beta = 90^\circ - \alpha,$$

$$\cos \beta = \cos(90^\circ - \alpha) = \sin \alpha,$$

$$\overline{DE} = a \cos \alpha + b \sin \alpha \quad (*)$$

$$X_{A1} = X_{A2} \Rightarrow F_B = F_C \quad (**)$$

$$Y_A = Y_{A1} + Y_{A2} = P_1 + P_2$$

$\overline{AB}$ :

$$\Sigma M_B = 0: P_1 \frac{a}{2} \cos \alpha - Y_{A1} a \cos \alpha + X_{A1} a \sin \alpha = 0;$$

$$\Sigma X_i = 0: F_B - X_{A1} = 0 \Rightarrow F_B = X_{A1}$$

$$\Sigma Y_i = 0: Y_{A1} - P_1 = 0 \Rightarrow Y_{A1} = P_1$$

$$\Sigma M_A = 0: F_B a \sin \alpha - P_1 \frac{a}{2} \cos \alpha = 0,$$

$$F_B = P_1 \frac{\cos \alpha}{2 \sin \alpha} = X_{A1}$$

$$(**) \Rightarrow P_1 \frac{\cos \alpha}{2 \sin \alpha} = P_2 \frac{\cos \beta}{2 \sin \beta}$$

$$P_1 \operatorname{tg} \beta = P_2 \operatorname{tg} \alpha; \quad \alpha + \beta = 90^\circ \Rightarrow P_1 \operatorname{ctg} \alpha = P_2 \operatorname{tg} \alpha,$$

$$\frac{1}{\operatorname{tg}^2 \alpha} = \frac{P_2}{P_1} \Rightarrow \operatorname{tg} \alpha = \sqrt{\frac{P_1}{P_2}}$$

$$\cos \alpha = \frac{1}{\sqrt{1 + \operatorname{tg}^2 \alpha}} = \frac{1}{\sqrt{1 + \frac{P_1}{P_2}}} = \frac{\sqrt{P_2}}{\sqrt{P_1 + P_2}}$$

$$\sin \alpha = \frac{\operatorname{tg} \alpha}{\sqrt{1 + \operatorname{tg}^2 \alpha}} = \frac{\sqrt{\frac{P_1}{P_2}}}{\sqrt{1 + \frac{P_1}{P_2}}} = \frac{\sqrt{P_1}}{\sqrt{P_1 + P_2}}$$

$$(*) \Rightarrow \overline{DE} = a \frac{\sqrt{P_2}}{\sqrt{P_1 + P_2}} + b \frac{\sqrt{P_1}}{\sqrt{P_1 + P_2}} = \frac{a\sqrt{P_2} + b\sqrt{P_1}}{\sqrt{P_1 + P_2}}$$

$\overline{AC}$ :

$$\Sigma M_C = 0: P_2 \frac{b}{2} \cos \beta - Y_{A2} b \cos \beta + X_{A2} b \sin \beta = 0$$

$$\Sigma X_i = 0: F_C - X_{A2} = 0 \Rightarrow F_C = X_{A2}$$

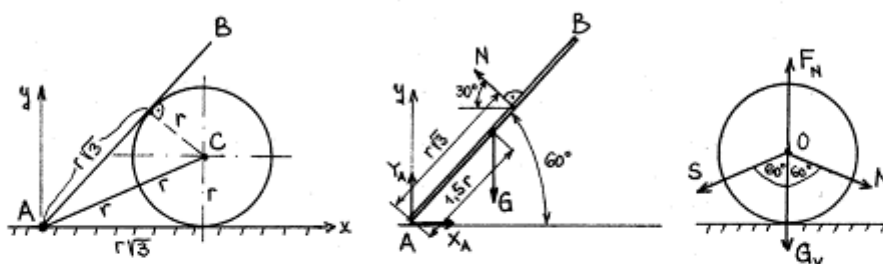
$$\Sigma Y_i = 0: Y_{A2} - P_2 = 0 \Rightarrow Y_{A2} = P_2$$

$$\Sigma M_A = 0: F_C b \sin \beta - P_2 \frac{b}{2} \cos \beta = 0,$$

$$F_C = P_2 \frac{\cos \beta}{2 \sin \beta} = X_{A2}$$

**ZADATAK BR. 23**

Homogeni prizmatični štap  $\overline{AB}$ , koji se može obrtati oko horizontalne ose  $A$ , oslanja se na omotač glatkog valjka, prečnika  $2r$ . Valjak leži na horizontalnoj ravni, a vezan je pomoću kanapa  $\overline{AC}$  za tačku  $A$ . Težina štapa iznosi  $16 \text{ kN}$ . Dužina štapa  $\overline{AB} = 3r$ , a kanapa  $\overline{AC} = 2r$ .  
 Odrediti silu  $S$  u koncu, pritisak štapa na kuglu i otpor zgloba  $A$ .



$$1) \sum X_i = 0 : X_A - N \cos 30^\circ = 0$$

$$2) \sum Y_i = 0 : Y_A - G + N \sin 30^\circ = 0$$

$$3) \sum M_A = 0 : N \cdot r\sqrt{3} - G \cdot 1,5r \cos 60^\circ = 0 \Rightarrow N = G \frac{1,5r}{r\sqrt{3}} \cdot \frac{1}{2} = \frac{3}{4} \frac{G}{\sqrt{3}}$$

$$N = G \frac{\sqrt{3}}{4} = 4\sqrt{3} \text{ [kN]}$$

$$1) \Rightarrow X_A = N \cos 30^\circ = G \frac{\sqrt{3}}{4} \cdot \frac{\sqrt{3}}{2} ;$$

$$X_A = \frac{3}{8} G = 6 \text{ [kN]}$$

$$2) \Rightarrow Y_A = G - N \sin 30^\circ = G - G \frac{\sqrt{3}}{4} \cdot \frac{1}{2} ;$$

$$Y_A = G \frac{(8-\sqrt{3})}{8} = 12,54 \text{ [kN]}$$

$$F_A = \sqrt{X_A^2 + Y_A^2} \Rightarrow$$

$$F_A = 13,9 \text{ [kN]}$$

$$\text{tg } \alpha = \frac{Y_A}{X_A} = \frac{12,54}{6} = 2,09 \Rightarrow$$

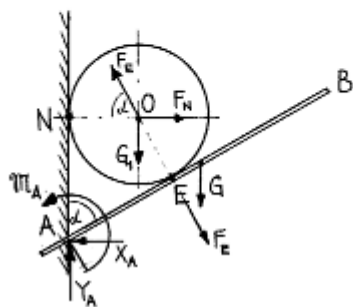
$$\alpha = 64,4^\circ$$

$$S \sin 60^\circ = N \sin 60^\circ \Rightarrow S = N = 4\sqrt{3} \text{ [kN]}$$

$$F_N = G_V + 2N \cos 60^\circ = G_V + N$$

**ZADATAK BR. 24**

Greda  $\overline{AB}$ , dužine  $l=0,8\text{ m}$  i težine  $G=100\text{ kN}$ , uzidana je u zid i to pod uglom od  $\alpha=60^\circ$ . Na njoj leži cilindar težine  $G_1=180\text{ kN}$ , koji dodiruje gredu u tački  $E$ , pri čemu je  $\overline{AE}=a=0,3\text{ m}$ .  
Odrediti reakcije veza.



CILINDAR:

$$1) \sum X_i = 0 : F_N - F_E \cos \alpha = 0 \Rightarrow F_N = \frac{1}{2} F_E$$

$$2) \sum Y_i = 0 : F_E \sin \alpha - G_1 = 0 \Rightarrow F_E = \frac{G_1}{\sin \alpha} = \frac{2\sqrt{3}}{3} G_1$$

$$\underline{F_E = 207,85 \text{ [kN]}} ; \underline{F_N = 103,92 \text{ [kN]}}$$

GREDA  $\overline{AB}$ :

$$1) \sum X_i = 0 : F_E \cos \alpha - X_A = 0 \Rightarrow X_A = \frac{G_1}{\sin \alpha} \cdot \cos \alpha = G_1 \operatorname{ctg} \alpha ; \underline{X_A = 103,92 \text{ [kN]}}$$

$$2) \sum Y_i = 0 : Y_A - F_E \sin \alpha - G = 0 \Rightarrow Y_A = G + F_E \sin \alpha = G + \frac{G_1}{\sin \alpha} \cdot \sin \alpha ,$$

$$Y_A = G + G_1 ; \underline{Y_A = 280 \text{ [kN]}}$$

$$\underline{F_A = \sqrt{X_A^2 + Y_A^2} = 298,67 \text{ [kN]}}$$

$$3) \sum M_A = 0 : M_A - F_E \cdot a - G \frac{l}{2} \sin \alpha = 0$$

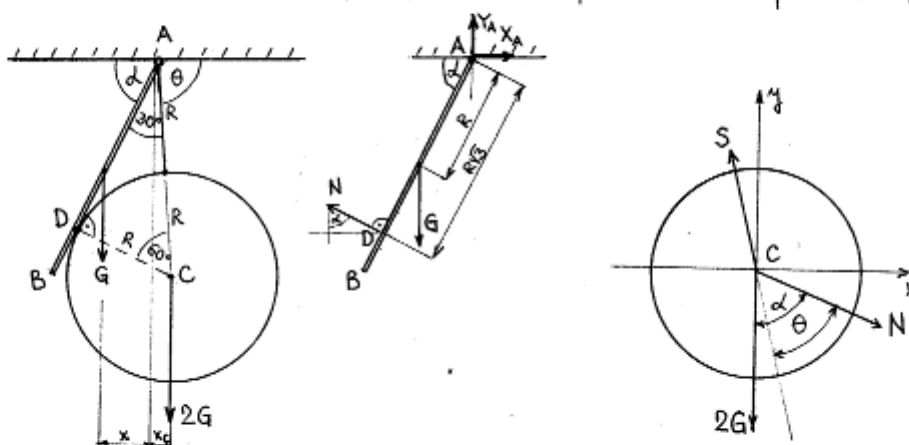
$$M_A = G_1 \frac{a}{\sin \alpha} + G \frac{l}{2} \sin \alpha$$

$$\underline{M_A \approx 97 \text{ [kNm]}}$$

**ZADATAK BR. 25**

Homogeni štap  $\overline{AB}$ , težine  $G$ , dužine  $2R$ , zglobno je vezan u tački  $A$ , a u tački  $D$  oslanja se o glatku kuglu, poluprečnika  $R$  i težine  $2G$ . Kugla je vezana nerastegljivim užetom dužine  $R$  za tačku  $A$ .

Odrediti ugao  $\alpha$  koji gradi štap sa horizontalnom osom u ravnotežnom položaju, reakciju zgloba  $A$  i pritisak štapa na kuglu.



$$1) \sum X_i = 0: X_A - N \sin \alpha = 0$$

$$2) \sum Y_i = 0: Y_A - G + N \cos \alpha = 0$$

$$3) \sum M_A = 0: N \cdot R\sqrt{3} - G R \cos \alpha = 0 \Rightarrow N = \frac{G\sqrt{3}}{3} \cos \alpha$$

$$1) \Rightarrow X_A = N \sin \alpha \Rightarrow X_A = \frac{G\sqrt{3}}{3} \sin \alpha \cos \alpha$$

$$2) \Rightarrow Y_A = G - N \cos \alpha = G - \frac{G\sqrt{3}}{3} \cos^2 \alpha = G \left(1 - \frac{\sqrt{3}}{3} \cos^2 \alpha\right)$$

$$\sum M_A = 0: 2G x_c - G x = 0 \Rightarrow 2x_c = x$$

$$x = R \cos \alpha \Rightarrow x_c = \frac{1}{2}x = \frac{1}{2}R \cos \alpha (*)$$

$$x_c = 2R \cos \theta = 2R \cos [180^\circ - (\alpha + 30^\circ)] = -2R \cos (\alpha + 30^\circ) =$$

$$= -2R (\cos \alpha \cos 30^\circ - \sin \alpha \sin 30^\circ) = -R (\sqrt{3} \cos \alpha - \sin \alpha)$$

$$x_c = R (\sin \alpha - \sqrt{3} \cos \alpha)$$

$$(*) \Rightarrow \frac{1}{2} R \cos \alpha = R (\sin \alpha - \sqrt{3} \cos \alpha)$$

$$\sin \alpha = \left(\frac{1}{2} + \sqrt{3}\right) \cos \alpha / : \cos \alpha$$

$$\operatorname{tg} \alpha = \frac{1}{2} + \sqrt{3} \Rightarrow \alpha = \operatorname{arctg} \left(\frac{1}{2} + \sqrt{3}\right) \Rightarrow \underline{\underline{\alpha = 65^\circ 5' = 65,84^\circ}}$$

$$1) \sum X_i = 0: N \sin \theta - S \cos \theta = 0$$

$$2) \sum Y_i = 0: S \sin \theta - N \cos \theta - 2G = 0$$

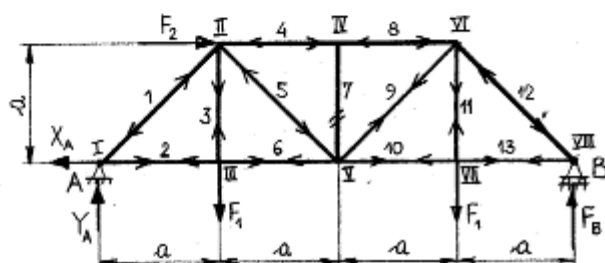
$$1) \Rightarrow S = N \frac{\sin \theta}{\cos \theta};$$

$$\underline{\underline{\theta = 180^\circ - (\alpha + 30^\circ)}}$$

**ZADATAK BR. 26**

Za rešetkasti nosač prikazan na slici

- analitičkom metodom odrediti otpore oslonaca i sile u štapovima;
- metodom Kremone odrediti veličinu i karakter sila u štapovima rešetke;
- metodom Ritera odrediti veličinu i karakter sila u štapovima u preseku R-R rešetke;



$$F_1 = 40 \text{ [kN]}, F_2 = 60 \text{ [kN]}, \\ a = 2 \text{ [m]}$$

a) Analitički metod određivanja otpora oslonaca

$$1) \sum X_i = 0 : -X_A + F_2 = 0 \Rightarrow X_A = F_2 = 60 \text{ [kN]}$$

$$2) \sum Y_i = 0 : Y_A - 2F_1 + F_B = 0$$

$$3) \sum \overset{\circ}{M}_B = 0 : Y_A \cdot 4a - F_1 \cdot 3a - F_1 \cdot a + F_2 \cdot a = 0 \quad / : a$$

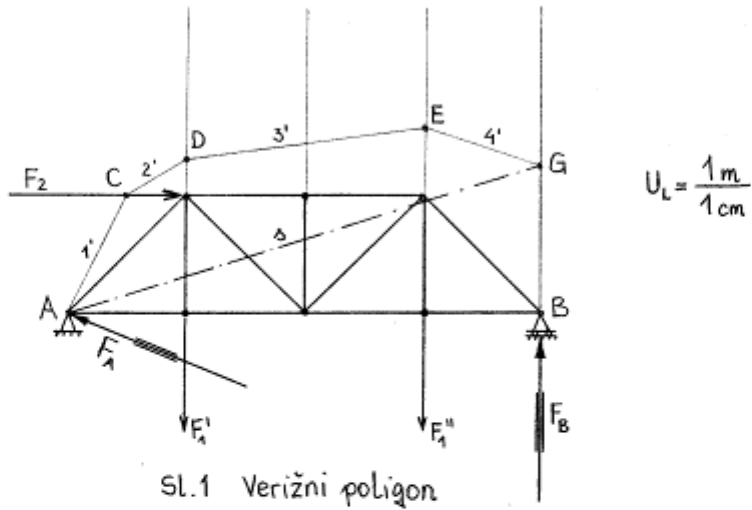
$$Y_A = F_1 - \frac{F_2}{4} = 40 - \frac{60}{4} \Rightarrow Y_A = 25 \text{ [kN]}$$

$$F_A = \sqrt{X_A^2 + Y_A^2} \Rightarrow F_A = 65 \text{ [kN]}$$

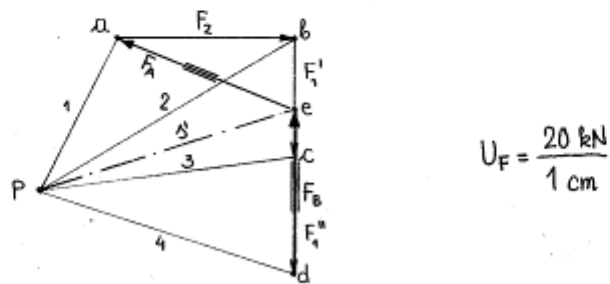
$$\alpha = \arctg \frac{Y_A}{-X_A} = \arctg \left( -\frac{5}{12} \right)$$

$$2) \Rightarrow F_B = 2F_1 - Y_A ; F_B = 55 \text{ [kN]}$$

\* Grafčki metod odreditvanja otpora oslonaca



Sl.1 Verižni poligon



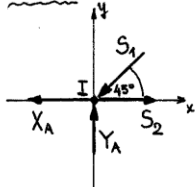
Sl.2 Plan (poligon) sila

$$F_A = U_F \cdot \bar{ea} = 65 \text{ [kN]}$$

$$F_B = U_F \cdot \bar{de} = 55 \text{ [kN]}$$

- \* Određivanje unutrašnjih sila (sila u štapovima rešetke);  
 - analitička metoda ravnoteže čvorova -

čvor I



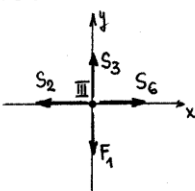
$$1) \sum X_i = 0 : -X_A - S_1 \cos 45^\circ + S_2 = 0$$

$$2) \sum Y_i = 0 : Y_A - S_1 \sin 45^\circ = 0$$

$$S_1 = \frac{Y_A}{\sin 45^\circ} \Rightarrow S_1 = 25\sqrt{2} \text{ [kN]}$$

$$S_2 = X_A + S_1 \cos 45^\circ = 60 + 25\sqrt{2} \frac{\sqrt{2}}{2} \Rightarrow S_2 = 85 \text{ [kN]}$$

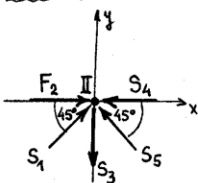
čvor III



$$1) \sum X_i = 0 : -S_2 + S_6 = 0 \Rightarrow S_6 = S_2 = 85 \text{ [kN]}$$

$$2) \sum Y_i = 0 : S_3 - F_1 = 0 \Rightarrow S_3 = F_1 = 40 \text{ [kN]}$$

čvor II



$$1) \sum X_i = 0 : F_2 + S_1 \cos 45^\circ - S_4 - S_5 \cos 45^\circ = 0$$

$$F_2 + S_1 \frac{\sqrt{2}}{2} - S_4 - S_5 \frac{\sqrt{2}}{2} = 0$$

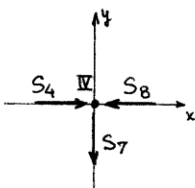
$$S_4 + S_5 \frac{\sqrt{2}}{2} = 85$$

$$2) \sum Y_i = 0 : S_1 \sin 45^\circ + S_5 \sin 45^\circ - S_3 = 0$$

$$S_5 = \frac{S_3}{\sin 45^\circ} - S_1 = 40\sqrt{2} - 25\sqrt{2} \Rightarrow S_5 = 15\sqrt{2} \text{ [kN]}$$

$$1) \Rightarrow S_4 = 85 - 15\sqrt{2} \frac{\sqrt{2}}{2} \Rightarrow S_4 = 70 \text{ [kN]}$$

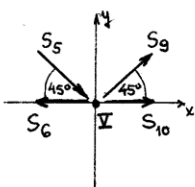
čvor IV



$$1) \sum X_i = 0 : S_4 - S_8 = 0 \Rightarrow S_8 = S_4 = 70 \text{ [kN]}$$

$$2) \sum Y_i = 0 : -S_7 = 0 \Rightarrow S_7 = 0$$

čvor V



$$1) \sum X_i = 0 : -S_6 + S_5 \cos 45^\circ + S_9 \cos 45^\circ + S_{10} = 0$$

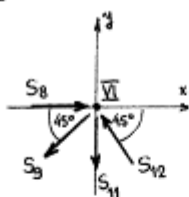
$$-85 + 15 + S_9 \frac{\sqrt{2}}{2} + S_{10} = 0$$

$$S_9 \frac{\sqrt{2}}{2} + S_{10} = 70$$

$$2) \sum Y_i = 0 : -S_5 \sin 45^\circ + S_9 \sin 45^\circ = 0 \Rightarrow S_9 = S_5 = 15\sqrt{2} \text{ [kN]}$$

$$1) \Rightarrow S_{10} = 70 - 15\sqrt{2} \frac{\sqrt{2}}{2} \Rightarrow S_{10} = 55 \text{ [kN]}$$

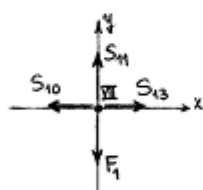
čvor VI



$$\begin{aligned}
 1) \sum X_i = 0 : S_8 - S_9 \cos 45^\circ - S_{12} \cos 45^\circ &= 0 \\
 S_2 \frac{\sqrt{2}}{2} &= S_8 - S_9 \frac{\sqrt{2}}{2} \\
 S_{12} &= S_8 \sqrt{2} - S_9 = 70\sqrt{2} - 15\sqrt{2} \\
 \underline{S_{12} = 55\sqrt{2} \text{ [kN]}}
 \end{aligned}$$

$$\begin{aligned}
 2) \sum Y_i = 0 : -S_9 \sin 45^\circ - S_{11} + S_{12} \sin 45^\circ &= 0 \\
 S_{11} &= S_{12} \frac{\sqrt{2}}{2} - S_9 \frac{\sqrt{2}}{2} = 55\sqrt{2} \frac{\sqrt{2}}{2} - 15\sqrt{2} \frac{\sqrt{2}}{2} \\
 \underline{S_{11} = 40 \text{ [kN]}}
 \end{aligned}$$

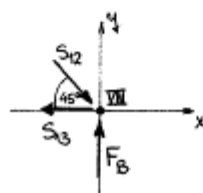
čvor VII



$$1) \sum X_i = 0 : -S_{10} + S_{13} = 0 \Rightarrow \underline{S_{13} = S_{10} = 55 \text{ [kN]}}$$

$$2) \sum Y_i = 0 : S_{11} - F_1 = 0 \Rightarrow \underline{S_{11} = F_1 = 40 \text{ [kN]}}$$

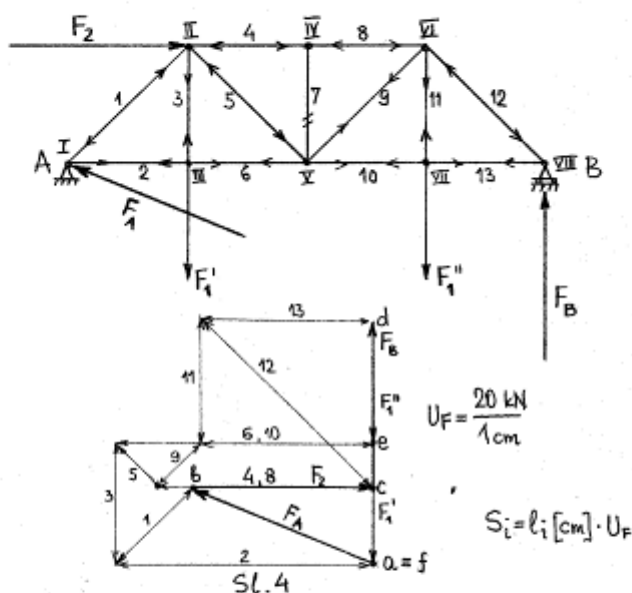
čvor VIII - PROVERA



$$\begin{aligned}
 1) \sum X_i = 0 : S_{12} \cos 45^\circ - S_{13} &= 0 \\
 55\sqrt{2} \frac{\sqrt{2}}{2} - 55 &= 0 \quad \checkmark
 \end{aligned}$$

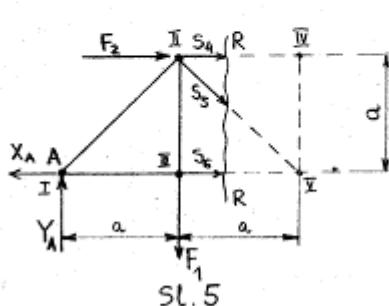
$$\begin{aligned}
 2) \sum Y_i = 0 : F_8 - S_{12} \sin 45^\circ &= 0 \\
 55 - 55\sqrt{2} \frac{\sqrt{2}}{2} &= 0 \quad \checkmark
 \end{aligned}$$

b) Kremonina metoda (grafička) odsecanja čvorova



i	Si [kN]	
	+	-
1		35
2	85	
3	40	
4		70
5		21
6	85	
7	/	/
8		70
9	21	
10	55	
11	40	
12	78	
13	55	

c) Riterova (grafoanalitička) metoda preseka



$$\sum \overset{\curvearrowright}{M}_I = 0: Y_A \cdot a + X_A \cdot a - S_6 \cdot a = 0$$

$$S_6 = Y_A + X_A = 85 \text{ [kN]}$$

$$\sum \overset{\curvearrowright}{M}_{II} = 0: Y_A \cdot 2a + F_2 \cdot a + S_4 \cdot a - F_1 \cdot a = 0$$

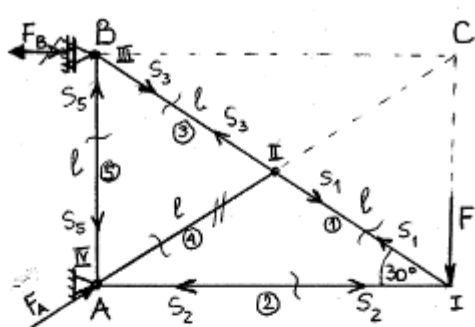
$$-S_4 = 2Y_A + F_2 - F_1 = 70 \text{ [kN]} \Rightarrow S_4 = -70 \text{ [kN]}$$

$$\sum \overset{\curvearrowright}{M}_{III} = 0: Y_A \cdot a + F_2 \cdot a + S_4 \cdot a + S_5 \frac{a\sqrt{2}}{2} = 0$$

$$S_5 \frac{\sqrt{2}}{2} = -Y_A - F_2 - S_4 = -15 \Rightarrow S_5 = -15\sqrt{2} \text{ [kN]}$$

## ZADATAK BR. 27

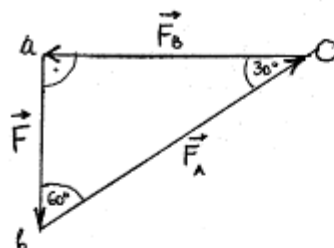
Odrediti sile u štapovima date rešetke opterećene silom  $F$  kao na slici.



$s$  - br. štapova  
 $n$  - br. čvorova

$$s = 2n - 3$$

$$s = 2 \cdot 4 - 3 \rightarrow s = 5$$

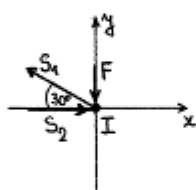


$$\frac{F}{\sin 30^\circ} = \frac{F_A}{\sin 90^\circ} = \frac{F_B}{\sin 60^\circ}$$

$$F_A = F \frac{\sin 90^\circ}{\sin 30^\circ} \Rightarrow F_A = 2F$$

$$F_B = F \frac{\sin 60^\circ}{\sin 30^\circ} \Rightarrow F_B = \sqrt{3}F$$

I čvor



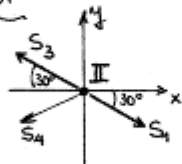
$$1) \sum X_i = -S_1 \cos 30^\circ + S_2 = 0$$

$$S_2 = S_1 \frac{\sqrt{3}}{2}$$

$$2) \sum Y_i = S_1 \sin 30^\circ - F = 0$$

$$S_1 = 2F; \quad S_2 = F\sqrt{3}$$

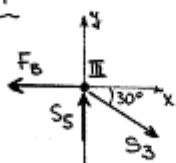
II čvor



$$\left. \begin{aligned} 1) \sum X_i &= S_1 \cos 30^\circ - S_3 \cos 30^\circ - S_4 \cos 30^\circ = 0 \\ 2) \sum Y_i &= -S_1 \sin 30^\circ + S_3 \sin 30^\circ - S_4 \sin 30^\circ = 0 \end{aligned} \right\} +$$

$$2S_4 = 0 \Rightarrow S_4 = 0; \quad S_3 = S_1 = 2F$$

III čvor



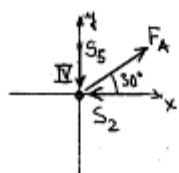
$$1) \sum X_i = S_3 \cos 30^\circ - F_B = 0$$

$$F_B = S_3 \frac{\sqrt{3}}{2} = 2F \frac{\sqrt{3}}{2} \Rightarrow F_B = F\sqrt{3}$$

$$2) \sum Y_i = S_5 - S_3 \sin 30^\circ = 0$$

$$S_5 = S_3 \cdot \frac{1}{2} \Rightarrow S_5 = F$$

IV čvor - PROVERA:



$$1) \sum X_i = F_A \cos 30^\circ - S_2 = 0$$

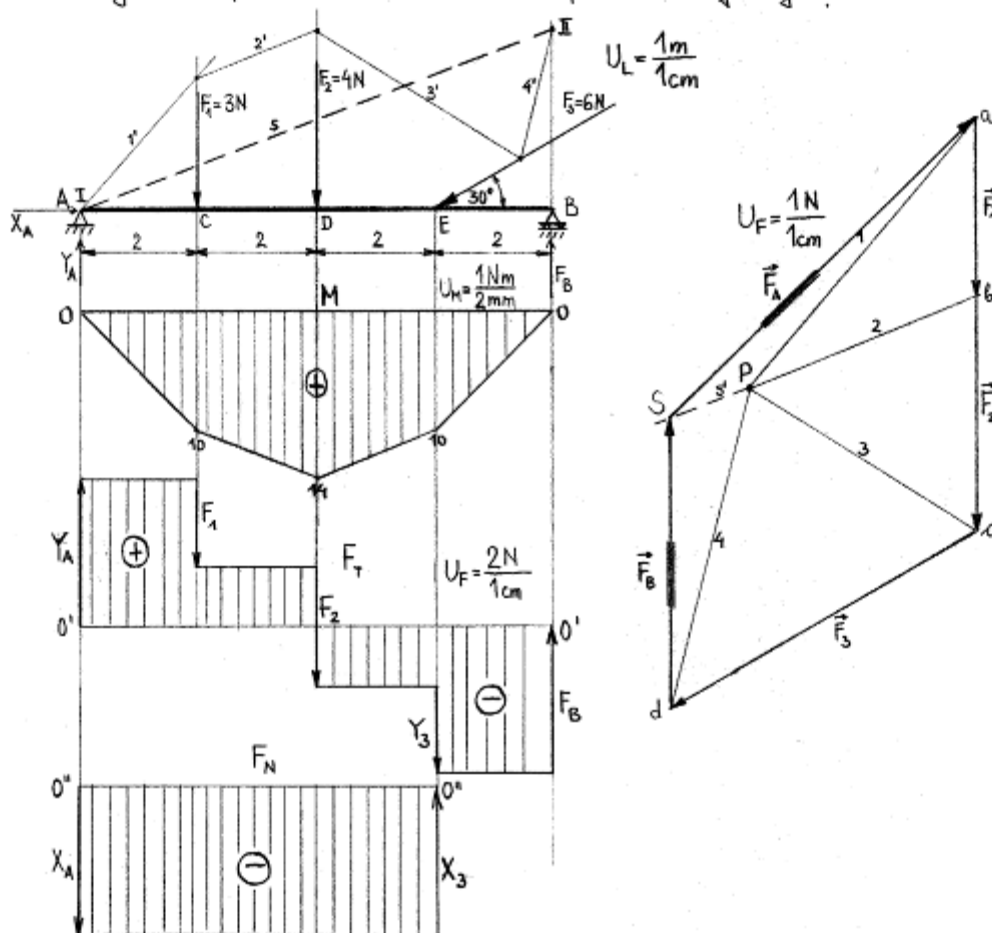
$$2F \frac{\sqrt{3}}{2} - F\sqrt{3} = 0 \Rightarrow 0 = 0 \quad \checkmark$$

$$2) \sum Y_i = F_A \sin 30^\circ - S_5 = 0$$

$$\frac{1}{2} 2F - F = 0 \Rightarrow 0 = 0 \quad \checkmark$$

## ZADATAK BR. 28

Prosta greda, raspona 8 [m], opterećena je datim teretima. Računski i grafički odrediti otpore oslonaca i nacrtati dijagrame napadnog momenta, transverzalne i aksijalne sile. Koliki je najveći napadni moment i u kom preseku dejstruje?



$$1) \sum X_i = 0; \quad X_A - F_3 \cos 30^\circ = 0 \Rightarrow X_A = F_3 \cos 30^\circ = 6 \frac{\sqrt{3}}{2} \Rightarrow X_A = 5,2 \text{ [N]}$$

$$2) \sum Y_i = 0; \quad Y_A - F_1 - F_2 - F_3 \sin 30^\circ + F_B = 0 \Rightarrow Y_A + F_B = 10$$

$$3) \sum M_B = 0; \quad Y_A \cdot 8 - F_1 \cdot 6 - F_2 \cdot 4 - F_3 \sin 30^\circ \cdot 2 = 0$$

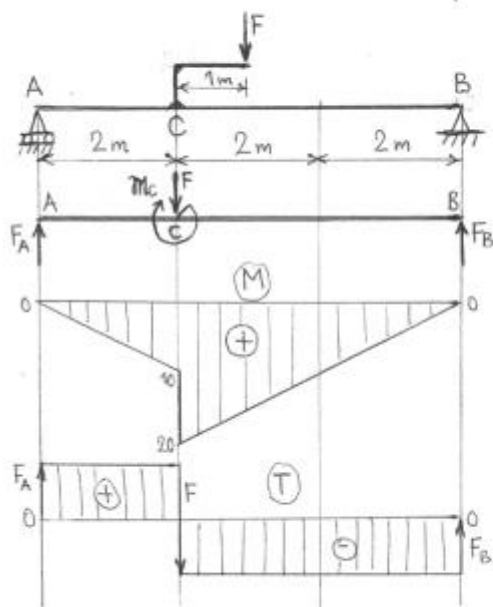
$$8Y_A = 6F_1 + 4F_2 + F_3 \Rightarrow Y_A = 5 \text{ [N]}; \quad F_B = 5 \text{ [N]}$$

$$F_A = \sqrt{X_A^2 + Y_A^2} \Rightarrow F_A = 7,2 \text{ [N]}; \quad \text{tg } \theta = \frac{Y_A}{X_A} \Rightarrow \theta = 43,88^\circ$$

$$M_C^l = Y_A \cdot 2 = 10 \text{ [Nm]}; \quad M_D^l = Y_A \cdot 4 - F_1 \cdot 2 = 14 \text{ [Nm]} = M_{\max}$$

**ZADATAK BR. 29**

Odrediti otpore oslonaca i nacrtati statičke dijagrame proste grede koja je u tački C ekscentrično opterećena vertikalnom silom intenziteta  $F=10$  [N].



$$M_C = F \cdot 1 = 10 \text{ Nm}$$

$$1) \sum X_i = 0$$

$$2) \sum Y_i = 0: F_A + F_B = 10$$

$$3) \sum M_A^{F_i} = 0: F_B \cdot 6 - F \cdot 2 - M_C = 0$$

$$6F_B = 2F + M_C$$

$$6F_B = 30$$

$$\underline{F_B = 5 \text{ N}}; \underline{F_A = 5 \text{ N}}$$

$$M_A = 0$$

$$M_{C,l}^t = F_A \cdot 2 = 10 \text{ Nm}$$

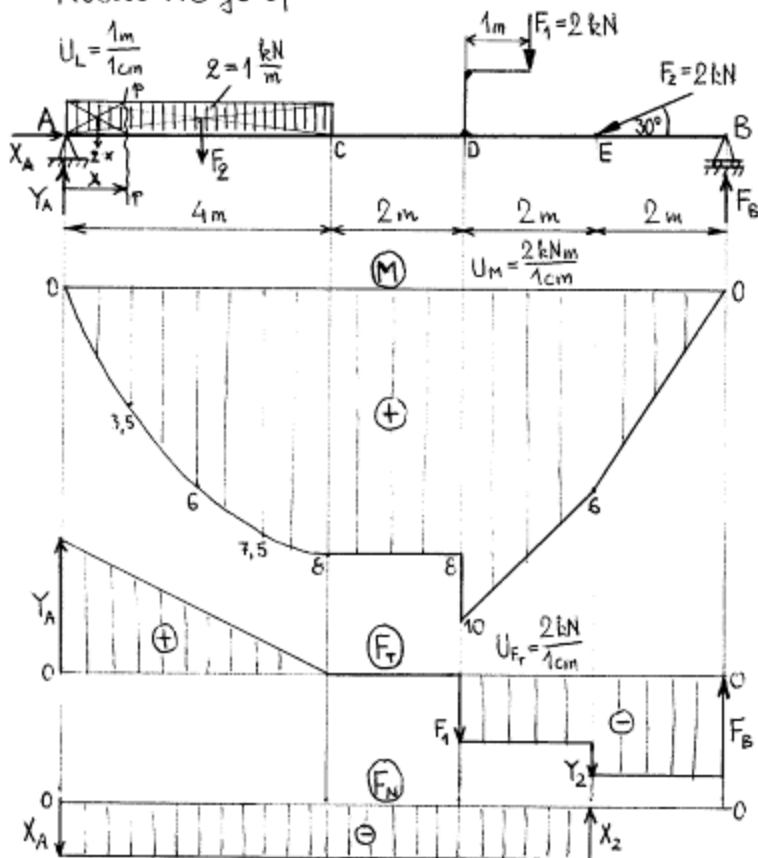
$$M_{C,d}^t = F_A \cdot 2 + M_C = 20 \text{ Nm}$$

$$M_{C,d}^d = F_B \cdot 4 = 20 \text{ Nm}$$

$$M_B = 0$$

**ZADATAK BR. 30**

Nosač AB je opterećen datim teretima. Nacrtati statičke dijagrame.



$$1) \sum X_i = 0 : X_A - F_2 \cos 30^\circ = 0 \Rightarrow X_A = 2 \cdot \frac{\sqrt{3}}{2} ; X_A = \sqrt{3} \text{ [kN]}$$

$$2) \sum Y_i = 0 : Y_A - F_2 - F_1 - F_2 \sin 30^\circ + F_B = 0 \Rightarrow Y_A + F_B = 2 \cdot 4 + 2 + 2 \cdot \frac{1}{2} ; F_2 = 2 \cdot 4 = 4 \text{ kN}$$

$$Y_A + F_B = 7$$

$$3) \sum \overset{\curvearrowright}{M}_B = 0 : Y_A \cdot 10 - F_2 \cdot 8 - F_1 \cdot 3 - F_2 \sin 30^\circ \cdot 2 = 0$$

$$10 Y_A = 32 + 6 + 2 = 40 \Rightarrow Y_A = 4 \text{ [kN]} , F_B = 3 \text{ [kN]}$$

$$F_A = \sqrt{X_A^2 + Y_A^2} = \sqrt{19} = 4,36 \text{ [kN]}$$

$$\theta_{F_A} = \arctg \frac{Y_A}{X_A} = 66,6^\circ$$

\* Napadni momenti u karakterističnim tačkama grede :

$$M_A = 0$$

$$M_C = Y_A \cdot 4 - F_2 \cdot 2 = 4 \cdot 4 - 4 \cdot 2 = 16 - 8 = 8 \text{ [kNm]}$$

$$M_D^l = Y_A \cdot 6 - F_2 \cdot 4 = 4 \cdot 6 - 4 \cdot 4 = 8 \text{ [kNm]}$$

$$M_D^d = Y_A \cdot 6 - F_2 \cdot 4 + M_D = 8 + F_1 \cdot 1 = 8 + 2 = 10 \text{ [kNm]}, \quad M_D = F_1 \cdot 1 = 2 \text{ [kNm]}$$

$$M_E = F_B \cdot 2 = 3 \cdot 2 = 6 \text{ [kNm]}$$

$$M_B = 0$$

+ Transverzalne sile u poljima :

$$A \div C : F_T = Y_A - 2 \cdot x \Big|_0^4 ; F_{TA} \Big|_{x=0} = Y_A = 4 \text{ [kN]} ; F_{TC} \Big|_{x=4m} = 0$$

$$D \div E : F_T = F_1 = -2 \text{ [kN]}$$

$$B \div E : F_T = F_B = 3 \text{ [kN]}$$

\* Napadni moment i transverzalna sila u preseku p-p :

$$\hat{M}_{p-p}(x) = Y_A \cdot x - 2 \cdot x \cdot \frac{1}{2}x = 4x - \frac{1}{2}x^2 ; \quad x \in [0, 4] \text{ m}$$

$$M(x=1m) = 3,5 \text{ [kNm]}$$

$$M(x=2m) = 6 \text{ [kNm]}$$

$$M(x=3m) = 7,5 \text{ [kNm]}$$

$$M(x=4m) = 8 \text{ [kNm]} = M_C$$

$$F_{T p-p}(x) = \frac{dM_{p-p}(x)}{dx} = 4 - x$$

$$F_T(x=0) = Y_A = 4 \text{ [kN]}$$

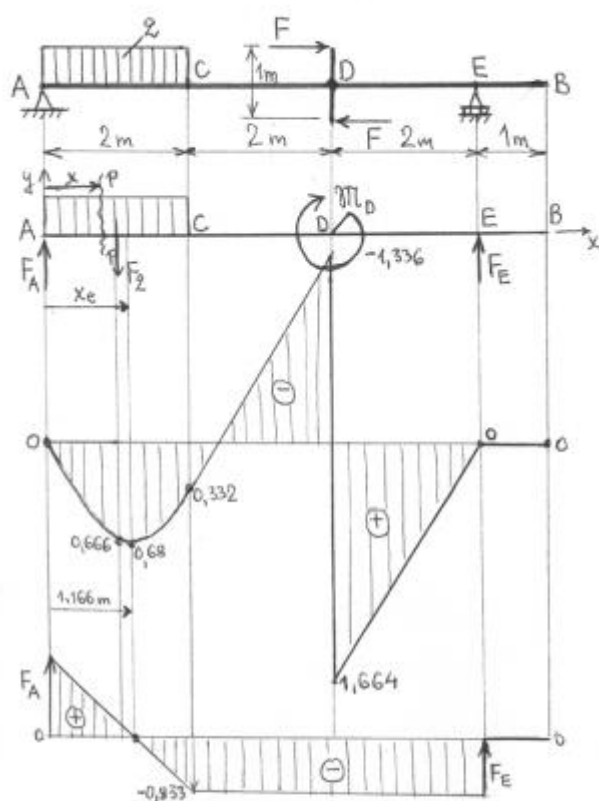
$$F_T(x=4m) = F_{TC} = 0$$

$$\frac{dM(x)}{dx} = 0 \Rightarrow 4 - x_e = 0 \Rightarrow \underline{x_e = 4m} ; \quad M(x_e) = M_{\max}(x) = 8 \text{ [kNm]}$$

\*  $M_{\max} = M_D = 10 \text{ [kNm]}$

**ZADATAK BR. 31**

Odrediti otpore oslonaca i nacrtati statičke dijagrame proste grede  $\bar{AB}$  koja je opterećena jednoliko raspodeljenim opterećenjem i spregom sila kao što je prikazano na slici.  $q = 1 \text{ [kN/m]}$ ,  $F = 3 \text{ kN}$ .



$$M_D = F \cdot 1 = 3 \text{ kNm}$$

$$F_2 = q \cdot 2 = 2 \text{ kN}$$

$$1) \sum X = 0$$

$$2) \sum Y = 0: F_A - F_2 + F_E = 0$$

$$F_A + F_E = 2$$

$$3) \sum M_A = 0: F_E \cdot 6 - M_D - F_2 \cdot 1 = 0$$

$$6F_E = M_D + F_2$$

$$6F_E = 5$$

$$F_E = \frac{5}{6} = 0,833 \text{ kN}$$

$$2) \Rightarrow F_A = 1,166 \text{ kN}$$

$$M_A = 0$$

$$M_C = F_A \cdot 2 - F_2 \cdot 1 = 0,332 \text{ kNm}$$

$$M_{D,c}^L = F_A \cdot 4 - F_2 \cdot 3 = -1,336 \text{ kNm}$$

$$M_{D,d}^L = F_A \cdot 4 - F_2 \cdot 3 + M_D = 1,664 \text{ kNm}$$

$$M_{D,d}^R = F_E \cdot 2 = 1,664 \text{ kNm}$$

$$M_E = 0$$

$$M(x) = F_A \cdot x - q \cdot x \cdot \frac{x}{2}; \quad 0 \leq x \leq 2 \text{ m}$$

$$M(x) = F_A x - \frac{1}{2} q x^2 = 1,166 x - 0,5 x^2$$

$$M(x=0) = M_A = 0$$

$$M(x=1) = 0,666 \text{ kNm}$$

$$M(x=2) = M_C = 0,332 \text{ kNm}$$

$$\frac{dM(x)}{dx} = 1,166 - x$$

$$\frac{dM(x)}{dx} = 0 \Rightarrow 1,166 - x_e = 0 \Rightarrow x_e = 1,166 \text{ m}$$

$$x_e \in [0, 2] \text{ m}$$

$$M(x_e) = 1,166 \cdot 1,166 - 0,5 (1,166)^2$$

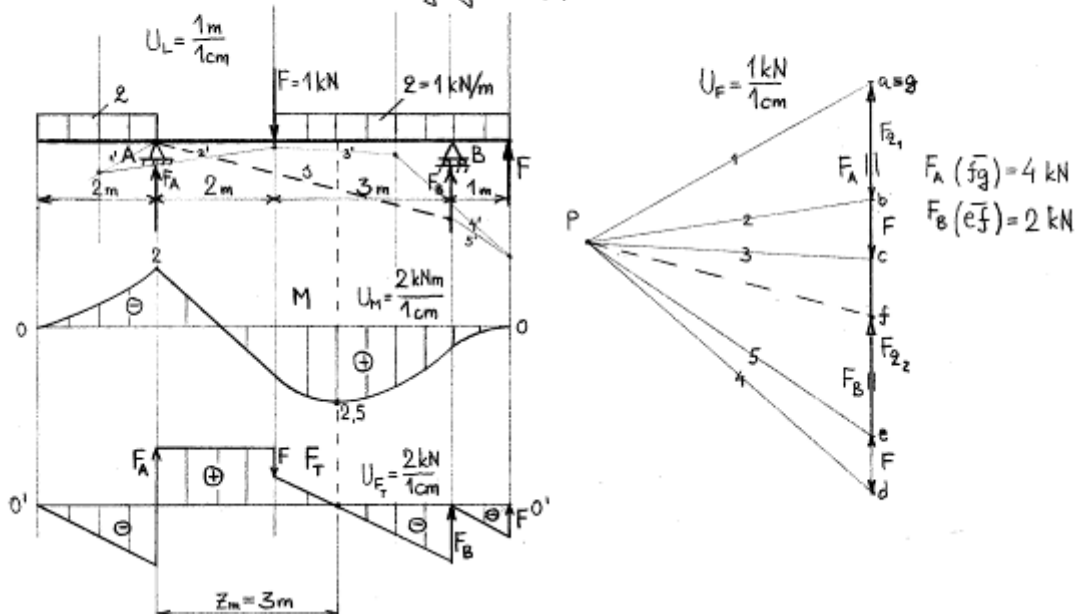
$$M(x_e) = 0,68 \text{ kNm}$$

$$M_{\max} = M_{D,d} = 1,664 \text{ kNm}$$

$$F_T(x) = \frac{dM(x)}{dx} = 1,166 - x; \quad 0 \leq x \leq 2 \text{ m}$$

**ZADATAK BR. 32**

Za prikazanu gredu sa prepustom, odrediti grafički i analitički otpore oslonaca, naći opasan presek i nacrtati statičke dijagrame.



$$1) \sum Y_i = 0 : F_A + F_B = 2 \cdot 2 + F + 2 \cdot 4 - F \Rightarrow F_A + F_B = 6$$

$$2) \sum M_A^d = 0 : 6F + 5F_B - 4 \cdot 2 \cdot 4 - 2F + 2 \cdot 2 \cdot 1 = 0 \Rightarrow 5F_B = 10 ; \underline{F_B = 2 \text{ kN}} , \underline{F_A = 4 \text{ kN}}$$

\* Određivanje opasnog preseka :

$$F_T = 0 : -2 + F_A - 1 - 2(z_m - 2) = 0 \Rightarrow \underline{z_m = 3 \text{ m}}$$

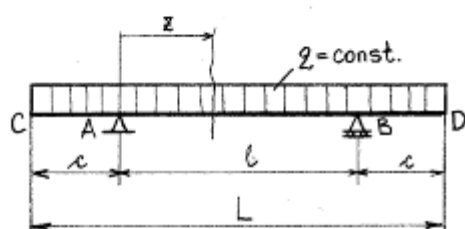
$$M(z_m) = M_{max} = F \cdot 3 + F_B \cdot 2 - 2 \cdot 3 \cdot 1,5 \Rightarrow \underline{M_{max} = 2,5 \text{ kNm}}$$

**ZADATAK BR. 33**

Greda ABCD, dužine  $L$ , sa dva jednaka prepusta  $c$ , opterećena je po celoj dužini jednolikim opterećenjem  $q = \text{const.}$

a) Na kom rastojanju  $l$  treba postaviti oslonce A i B da bi apsolutne vrednosti maksimalnih pozitivnih i negativnih napadnih momenata (tzv. granične vrednosti) bile jednake?

b) U kom odnosu, u tom slučaju, stoje dužine prepusta  $c$  sa rasponom oslonca  $l$ ?



$$L = l + 2c$$

$$F_A = F_B$$

$$F_2 = qL$$

$$a) \sum Y_i = 0 : F_A + F_B - qL = 0 \rightarrow F_A = F_B = \frac{1}{2} qL$$

$$M_{\min} = M_A = M_B = -\frac{1}{2} qc^2 ; \quad c = \frac{1}{2}(L-l), \quad c^2 = \frac{1}{4}(L-l)^2$$

$$M_{\min} = -\frac{1}{8} q(L-l)^2$$

$$\begin{aligned} M_{\max} = M\left(\frac{1}{2}l\right) &= -qc \frac{1}{2}(c+l) + \frac{1}{2}F_A \cdot l - q \frac{l}{2} \cdot \frac{l}{4} = \\ &= -\frac{1}{2}qc^2 - \frac{1}{2}qcl + \frac{1}{2} \cdot \frac{1}{2}qLl - \frac{1}{8}qL^2 = \\ &= -\frac{1}{2}q \frac{1}{4}(L-l)^2 - \frac{1}{2}q \frac{1}{2}(L-l)l + \frac{1}{4}qLl - \frac{1}{8}qL^2 = \\ &= -\frac{1}{8}q(L^2 - 2Ll + l^2) - \frac{1}{4}q(Ll - l^2) + \frac{1}{4}qLl - \frac{1}{8}qL^2 = \\ &= \frac{1}{8}qL(2l-L) \end{aligned}$$

$$|M_{\min}| = |M_{\max}| \Rightarrow \frac{1}{8}q(L-l)^2 = \frac{1}{8}qL(2l-L)$$

$$l^2 - 4Ll + 2L^2 = 0$$

$$l_{1/2} = \frac{4L \pm \sqrt{16L^2 - 8L^2}}{2} = 2L \pm L\sqrt{2}$$

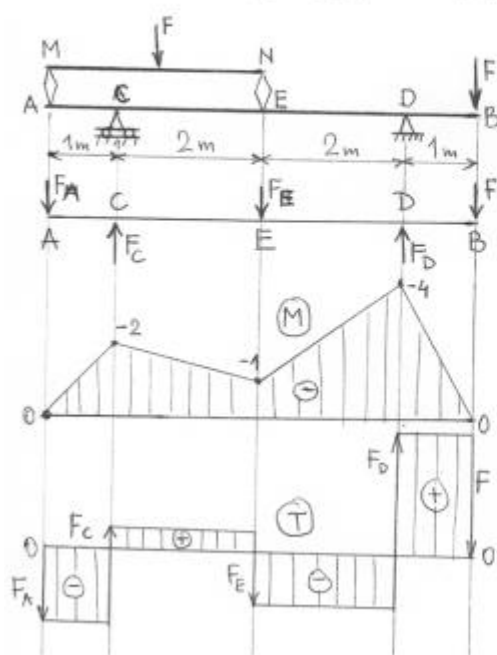
$$l < L \Rightarrow l = (2 - \sqrt{2})L$$

$$b) L = l + 2c \rightarrow l = (2 - \sqrt{2})(l + 2c)$$

$$\frac{c}{l} = 0,347$$

**ZADATAK BR. 34**

Odrediti otpore oslonaca C i D proste grede  $\overline{AB}$  posredno opterećene u tačkama A i E prostom gredom  $\overline{MN}$  na koju na sredini raspona dejstvuje sila F, kao i silom F u tački B.  $F=4 \text{ kN}$ .



$$\overline{MN}: F_M + F_N = F = 4 ; F_M = F_N = 2 \text{ kN}$$

$$F_A = F_M = 2 \text{ kN}; F_E = F_N = 2 \text{ kN}$$

$$\overline{AB}: 1) \sum X_i = 0$$

$$2) \sum Y_i = 0: F_C + F_D = 8$$

$$3) \sum M_C^F = 0: -F \cdot 5 + F_D \cdot 4 - F_E \cdot 2 + F_A \cdot 1 = 0$$

$$4F_D = 5F + 2F_E - F_A$$

$$4F_D = 20 + 4 - 2$$

$$4F_D = 22$$

$$F_D = 5,5 \text{ kN}; F_C = 2,5 \text{ kN}$$

$$M_A = 0$$

$$M_C = -F_A \cdot 1 = -2 \text{ kNm}$$

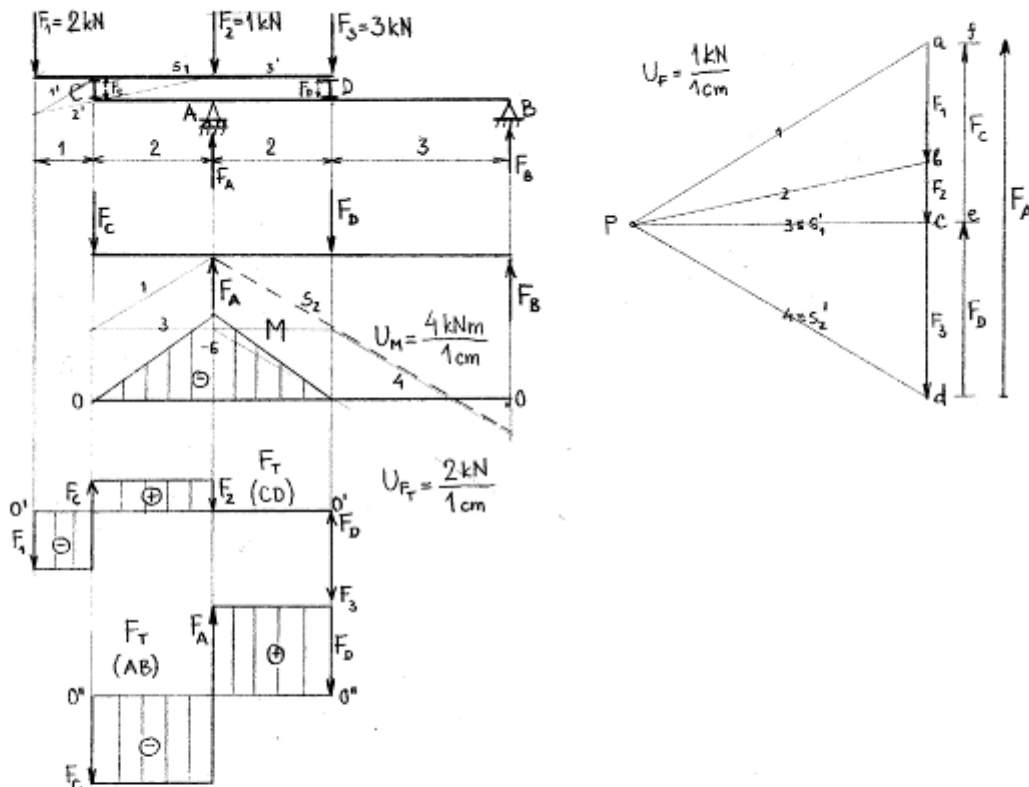
$$M_E = -F_A \cdot 3 + F_C \cdot 2 = -6 + 5 = -1 \text{ kNm}$$

$$M_D = -F \cdot 1 = -4 \text{ kNm}$$

$$M_B = 0$$

**ZADATAK BR. 35**

Za date opterećene grede,  $F_1 = 2 \text{ kN}$ ,  $F_2 = 1 \text{ kN}$ ,  $F_3 = 3 \text{ kN}$ , računski i grafički odrediti otpore oslonaca. Dijagram napadnog momenta nacrtati za obe grede ujedno.



Greda CD:  $\sum Y = 0$ ,  $F_C + F_D - F_1 - F_2 - F_3 = 0 \rightarrow F_C + F_D = 6$

$\sum M_D = 0$ ,  $-F_1 \cdot 5 + F_C \cdot 4 - F_2 \cdot 2 = 0 \rightarrow \underline{F_C = 3 \text{ kN}}$ ,  $\underline{F_D = 3 \text{ kN}}$

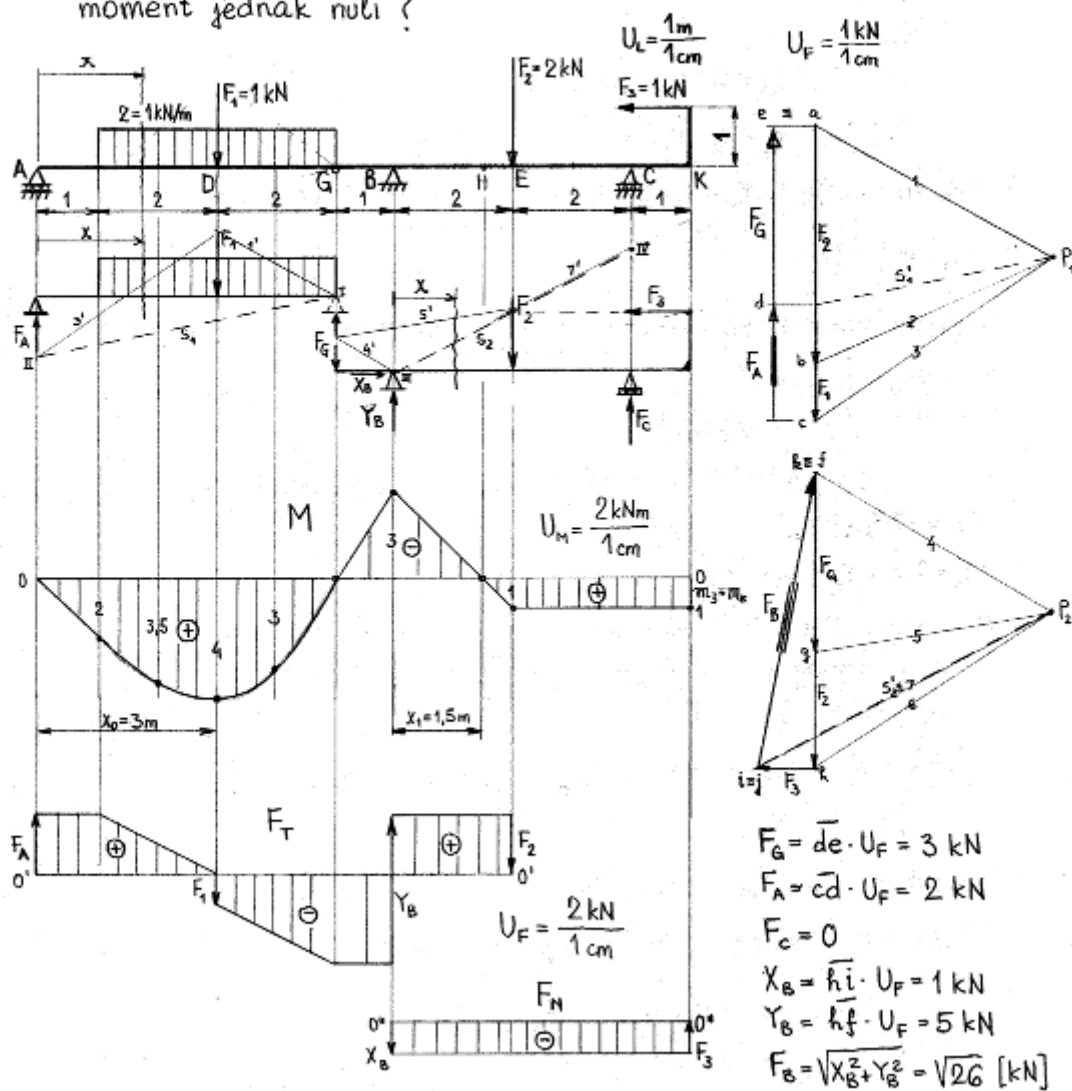
Greda AB:  $\sum Y = 0$ ,  $F_A + F_B = F_C + F_D \rightarrow F_A + F_B = 6$

$\sum M_B = 0$ ,  $-F_C \cdot 7 + F_A \cdot 5 - F_D \cdot 3 = 0 \rightarrow \underline{F_A = 6 \text{ kN}}$ ,  $\underline{F_B = 0}$

$M_A^l = -F_C \cdot 2 = -6 \text{ kNm}$

**ZADATAK BR. 36**

Za dati Gerberov nosač, opterećen teretima kao na slici, računski i grafički odrediti otpore oslonaca i nacrtati statičke dijagrame. Odrediti položaj opasnog preseka i izračunati brojnu vrednost najvećeg napadnog momenta. U kojim je presjecima napadni moment jednak nuli?



GREDA  $\overline{AG}$ :

$$1) \sum Y_i = 0 : F_A + F_G - F_1 - F_2 = 0 \Rightarrow F_A + F_G = 5$$

$$2) \sum M_A = 0 : 5F_G - 3(F_2 + F_1) = 0 \Rightarrow F_G = \frac{3}{5}(F_2 + F_1) ; \underline{F_G = 3 \text{ [kN]}}, \underline{F_A = 2 \text{ [kN]}}$$

$$3) M_G^l = M_G^d = 0$$

GREDA  $\overline{BC}$

$$1) \sum X_i = 0 : X_B - F_3 = 0 ; \underline{X_B = 1 \text{ [kN]}}$$

$$2) \sum Y_i = 0 : Y_B + F_C - F_G - F_2 = 0 \Rightarrow Y_B + F_C = 5$$

$$3) \sum M_C = 0 : -5F_G + 4Y_B - 2F_2 - F_3 = 0 \Rightarrow \underline{Y_B = 5 \text{ [kN]}}, \underline{F_C = 0}$$

$$4) M_G^l = M_G^d = 0$$

\* Napadni momenti na karakterističnim mestima :

$$M_A = M_G = 0$$

$$M_D = 3F_A - 2 \cdot 2 \cdot 1 = 4 \text{ [kNm]}$$

$$M_B = -F_G \cdot 1 = -3 \text{ [kNm]}$$

$$M_E = -3F_G + 2Y_B = 1 \text{ [kNm]}$$

$$M_C = M_K = F_3 \cdot 1 = 1 \text{ [kNm]}$$

$$\left. \begin{array}{l} M_A = M_G = 0 \\ M_D = 3F_A - 2 \cdot 2 \cdot 1 = 4 \text{ [kNm]} \\ M_B = -F_G \cdot 1 = -3 \text{ [kNm]} \\ M_E = -3F_G + 2Y_B = 1 \text{ [kNm]} \\ M_C = M_K = F_3 \cdot 1 = 1 \text{ [kNm]} \end{array} \right\} \Rightarrow M_{\max} = M_D = 4 \text{ [kNm]} \Rightarrow \text{opasan presek u tački D}$$

\* Analitičko određivanje najvećeg napadnog momenta :

J-na napadnog momenta u polju  $\overline{AD}$  :

$$M(x) = F_A \cdot x - \frac{1}{2} 2(x-1)^2$$

$$\frac{dM(x)}{dx} = F_A - 2(x-1) = 0 \Rightarrow \underline{x_D = 3 \text{ m}} ; x = 3 \text{ m} \Rightarrow F_T = 0$$

\* Momentna j-na u polju BE :

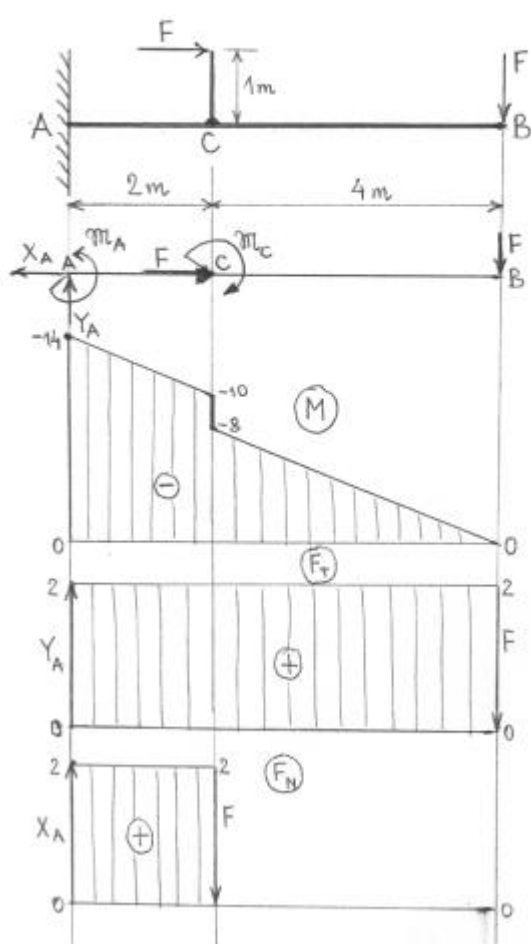
$$-F_G(x_1 + 1) + Y_B x_1 = 0 \Rightarrow \underline{x_1 = \overline{BH} = 1,5 \text{ m}}$$

$$M_H^l = F_A(6 + x_1) - (F_1 + F_2)(3 + x_1) + Y_B \cdot x_1 = 0$$

$$M_H^d = F_3 \cdot 1 - F_2(2 - x_1) = 0$$

**ZADATAK BR. 37**

Odrediti otpore uklještenja i nacrtati statičke dijagrame lakog konzolnog nosača  $\overline{AB}$  opterećenog koncentrisanom silom  $F$  na slobodnom kraju  $B$  i horizontalnom ekscentričnom silom  $F=2\text{ kN}$  kao što je prikazano na slici.



$$M_C = F \cdot 1 = 2 \text{ kNm}$$

$$1) \sum X_i = 0 : F - X_A = 0 \Rightarrow \underline{X_A = F = 2 \text{ kN}}$$

$$2) \sum Y_i = 0 : Y_A - F = 0 \Rightarrow \underline{Y_A = F = 2 \text{ kN}}$$

$$3) \sum M_A = 0 : -F \cdot 6 - M_C + M_A = 0$$

$$M_A = 6F + M_C = 12 + 2$$

$$\underline{M_A = 14 \text{ kNm}}$$

$$M_A = -M_A = -14 \text{ kNm}$$

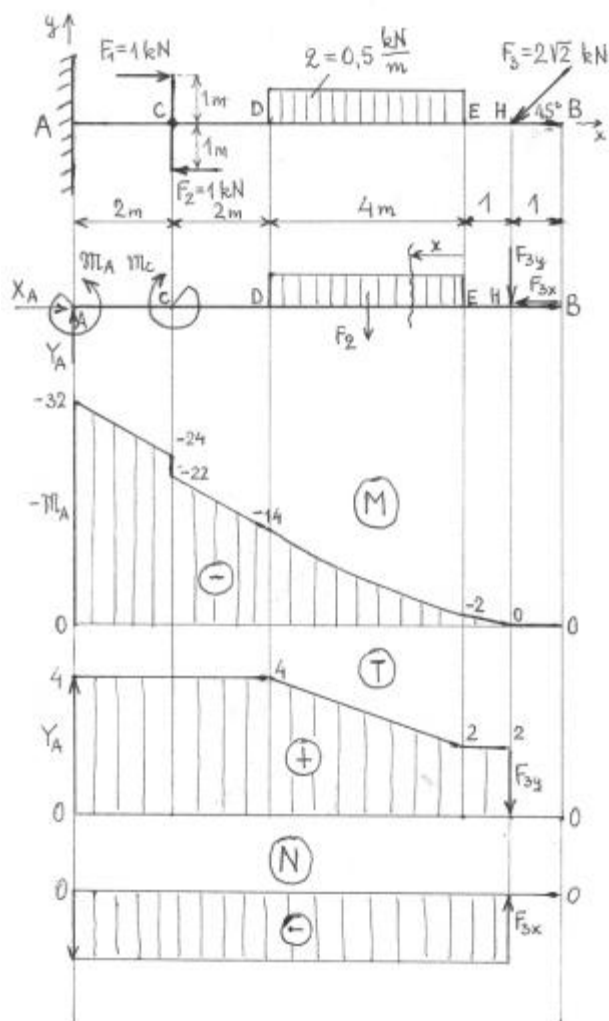
$$M_{C,C} = Y_A \cdot 2 - M_A = -10 \text{ kNm}$$

$$M_{C,d} = Y_A \cdot 2 - M_A + M_C = -8 \text{ kNm}$$

$$M_B = 0$$

**ZADATAK BR. 38**

Za laki konzolni nosač, dimenzija i opterećenja datih na slici, odrediti otpor uklještenja i nacrtati statičke dijagrame.



$$1) \sum X_i = 0 : X_A - F_{3x} = 0 ; F_{3x} = F_3 \cos 45^\circ = 2 \text{ kN}$$

$$\underline{X_A = F_{3x} = 2 \text{ kN}}$$

$$2) \sum Y_i = 0 : Y_A - F_2 - F_{3y} = 0 ; F_2 = q \cdot 4 = 2 \text{ kN}$$

$$F_{3y} = F_3 \sin 45^\circ = 2 \text{ kN}$$

$$Y_A = F_2 + F_{3y}$$

$$\underline{Y_A = 4 \text{ kN}}$$

$$3) \sum M_A = 0 : -F_{3y} \cdot 9 - F_2 \cdot 6 - M_C + M_A = 0 ;$$

$$M_C = F_1 \cdot 2 = F_2 \cdot 2 = 2 \text{ kNm}$$

$$M_A = M_C + 6F_2 + 9F_{3y}$$

$$M_A = 2 + 12 + 18$$

$$\underline{M_A = 32 \text{ kNm}}$$

$$M_A = -M_A = -32 \text{ kNm}$$

$$M_{C,l} = -M_A + Y_A \cdot 2 = -32 + 8 = -24 \text{ kNm}$$

$$M_{C,d} = -M_A + Y_A \cdot 2 + M_C = -22 \text{ kNm}, \text{ ili:}$$

$$M_{C,d} = -F_{3y} \cdot 7 - F_2 \cdot 4 = -14 - 8 = -22 \text{ kNm}$$

$$M_D = -F_{3y} \cdot 5 - F_2 \cdot 2 = -14 \text{ kNm}$$

$$M_E = -F_{3y} \cdot 1 = -2 \text{ kNm}$$

$$M_H = 0$$

$$M_B = 0$$

$$0 \leq x \leq 4 \text{ m:}$$

$$M(x) = -F_{3y}(1+x) - q \cdot \frac{1}{2} x^2 = -2 - 2x - \frac{1}{4} x^2$$

$$M(0) = M_E = -2 \text{ kNm}$$

$$M(4) = M_D = -14 \text{ kNm}$$

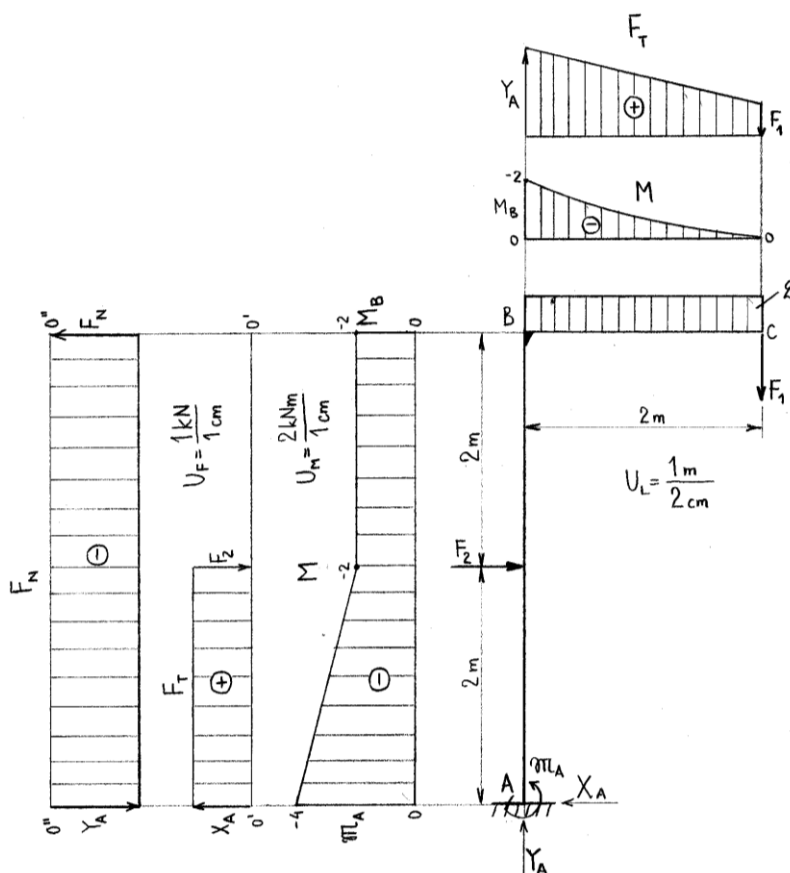
$$\frac{dM}{dx} = -2 - \frac{1}{2} x_e ; -2 - \frac{1}{2} x_e = 0 \Rightarrow x_e = -4 \text{ m} \notin [0,4]$$

$$F_T(x) = -\frac{dM(x)}{dx} = 2 + \frac{1}{2} x, \quad 0 \leq x \leq 4 \text{ m}$$

**ZADATAK BR. 39**

Konzola AB savijena je kod B pod pravim uglom i opterećena datim teretima. Odrediti otpore oslonaca i nacrtati statičke dijagrame ako su:

$$q = 0,5 \text{ kN/m}; F_1 = 0,5 \text{ kN}; F_2 = 1 \text{ kN}$$



$$1) \sum X_i = 0 : F_2 - X_A = 0 \Rightarrow X_A = 1 \text{ [kN]}$$

$$2) \sum Y_i = 0 : Y_A - F_2 - F_1 = 0 \Rightarrow Y_A = 1,5 \text{ [kN]}$$

$$3) \sum M_A = 0 : M_A - 2F_2 - 1 \cdot F_2 - 2F_1 = 0 \Rightarrow M_A = 4 \text{ [kNm]}$$

$$M_B = -2F_1 - 1F_2 = -2 \text{ [kNm]}$$



## TEŽIŠTA

Težište tela je ona njegova tačka koja pri ma kojem položaju tela ostaje nepromenjena, tj. ostaje napadna tačka njegove težine.

1. TEŽIŠTA LINIJA - Kada su dve dimenzije tela male prema trećoj, tako da ih možemo zanemariti, telo smatramo teškom linijom.

$$x_c = \frac{\sum_{i=1}^n L_i x_i}{L}; \quad y_c = \frac{\sum_{i=1}^n L_i y_i}{L}; \quad z_c = \frac{\sum_{i=1}^n L_i z_i}{L} \quad \text{- ako se homogeno telo može podeliti na konačan broj linija}$$

$$x_c = \frac{\int x dl}{L}; \quad y_c = \frac{\int y dl}{L}; \quad z_c = \frac{\int z dl}{L} \quad \text{- ako telo ne možemo podeliti na konačan broj linija}$$

2. TEŽIŠTA POVRŠINA - Kada je jedna dimenzija tela mala prema drugim dvema, tako da je možemo zanemariti, telo smatramo teškom površinom.

$$x_c = \frac{\sum_{i=1}^n A_i x_i}{A}; \quad y_c = \frac{\sum_{i=1}^n A_i y_i}{A}; \quad z_c = \frac{\sum_{i=1}^n A_i z_i}{A}; \quad A = \sum_{i=1}^n A_i$$

$$x_c = \frac{\iint x dA}{A}; \quad y_c = \frac{\iint y dA}{A}; \quad z_c = \frac{\iint z dA}{A}; \quad A = \iint_{(A)} dA$$

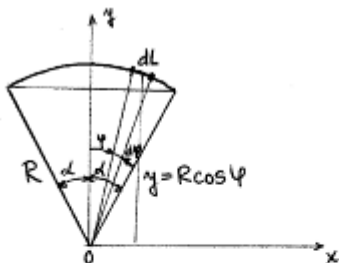
3. TEŽIŠTA ZAPREMINA

$$x_c = \frac{\iiint x dV}{V}; \quad y_c = \frac{\iiint y dV}{V}; \quad z_c = \frac{\iiint z dV}{V}$$

$$x_c = \frac{\sum_{i=1}^n x_i V_i}{V}; \quad y_c = \frac{\sum_{i=1}^n y_i V_i}{V}; \quad z_c = \frac{\sum_{i=1}^n z_i V_i}{V}$$

**ZADATAK BR. 41**

Odrediti koordinate težišta kružnog luka.



$$z_c = 0 ; x_c = 0$$

$$y_c = \frac{\int y dL}{\int dL}$$

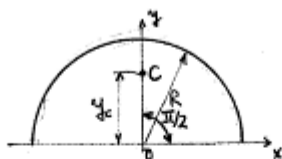
$$\Delta L = R d\varphi$$

$$L = \int dL = \int_{-\alpha}^{\alpha} R d\varphi = R \int_{-\alpha}^{\alpha} d\varphi = R \varphi \Big|_{-\alpha}^{\alpha} = R [\alpha - (-\alpha)]$$

$$L = 2R\alpha$$

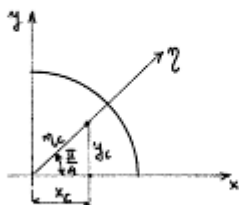
$$\int y dL = \int_{-\alpha}^{\alpha} R \cos \varphi \cdot R d\varphi = R^2 \int_{-\alpha}^{\alpha} \cos \varphi d\varphi = R^2 \sin \varphi \Big|_{-\alpha}^{\alpha} = R^2 [\sin \alpha - \sin(-\alpha)] = 2R^2 \sin \alpha$$

$$y_c = \frac{2R^2 \sin \alpha}{2R\alpha} \Rightarrow y_c = \frac{R \sin \alpha}{\alpha} \quad (\alpha \text{ u radijanima})$$



$$y_c = \frac{R \sin \frac{\pi}{2}}{\frac{\pi}{2}} = \frac{R \cdot 1}{\frac{\pi}{2}} \Rightarrow y_c = \frac{2R}{\pi}$$

$$x_c = 0$$



$$\alpha = \frac{\pi}{4}$$

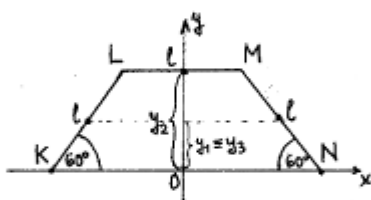
$$x_c = y_c = \eta_c \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} \eta_c$$

$$\eta_c = \frac{R \sin \frac{\pi}{4}}{\frac{\pi}{4}} = \frac{R \frac{\sqrt{2}}{2}}{\frac{\pi}{4}} \Rightarrow \eta_c = \frac{2\sqrt{2} R}{\pi}$$

$$x_c = y_c = \frac{\sqrt{2}}{2} \frac{2\sqrt{2} R}{\pi} \Rightarrow x_c = y_c = \frac{2R}{\pi}$$

**ZADATAK BR. 42**

Odrediti položaj težišta poligona KLMN, strane  $l = 6 \text{ cm}$ , koji predstavlja polovinu šestougona.



$$x_c = 0$$

$$y_c = \frac{\sum L_i y_i}{L}$$

$$L = \sum_{i=1}^3 L_i = 3l = 18 \text{ cm}$$

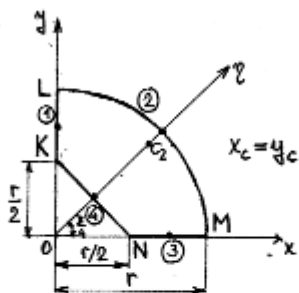
$$y_1 = y_3 = \frac{l}{2} \sin 60^\circ = \frac{l}{2} \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2} \text{ cm}$$

$$y_2 = l \cdot \sin 60^\circ = l \frac{\sqrt{3}}{2} = 3\sqrt{3} \text{ cm}$$

$$y_c = \frac{l \cdot y_1 + l \cdot y_2 + l \cdot y_3}{L} = \frac{l(2 \cdot y_1 + y_2)}{L} = \frac{6 \left( 2 \cdot \frac{3\sqrt{3}}{2} + 3\sqrt{3} \right)}{18} \Rightarrow \underline{y_c = 2\sqrt{3} \text{ cm}}$$

**ZADATAK BR. 43**

Odrediti koordinate težišta homogenog poligona KLMN.



$l$	$L_i$ [cm]	$y_i$ [cm]	$L_i y_i$ [cm <sup>2</sup> ]
1	$r/2$	$\frac{3}{4}r$	$\frac{3}{8}r^2$
2	$\pi r/2$	$\frac{2r}{\pi}$	$r^2$
3	$r/2$	0	0
4	$r\frac{\sqrt{2}}{2}$	$\frac{r}{4}$	$\frac{\sqrt{2}}{8}r^2$
$\Sigma$	$r\left(\frac{1}{2} + \frac{\pi}{2} + \frac{1}{2} + \frac{\sqrt{2}}{2}\right)$	/	$r^2\left(\frac{3}{8} + 1 + \frac{\sqrt{2}}{8}\right)$

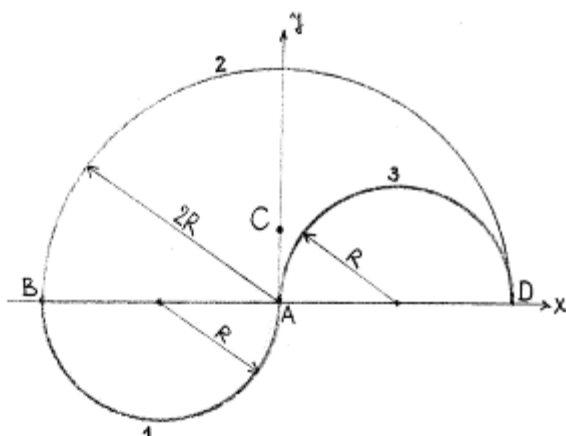
$$r_2 = \frac{r \sin d}{d} = \frac{r \frac{\sqrt{2}}{2}}{\frac{\pi}{4}} = \frac{2\sqrt{2}r}{\pi}$$

$$y_2 = r_2 \cos \frac{\pi}{4} = \frac{2\sqrt{2}r}{\pi} \frac{\sqrt{2}}{2} \Rightarrow y_2 = \frac{2r}{\pi}$$

$$y_c = x_c = \frac{\sum_{i=1}^4 L_i y_i}{\sum_{i=1}^4 L_i} = \frac{\frac{r^2}{8}(11 + \sqrt{2})}{\frac{r}{2}(2 + \sqrt{2} + \pi)} = \frac{r}{4} \frac{(11 + \sqrt{2})}{(2 + \sqrt{2} + \pi)} \approx 0,47r$$

**ZADATAK BR. 44**

Analitički odrediti položaj težišta zatvorene kružne linije ABDA, pri čemu je  $R = 4 \text{ cm}$ .



	$i$	$L_i$	$x_i$	$y_i$	$L_i x_i$	$L_i y_i$
$\widehat{AB}$	1	$R\pi$	$-R$	$-\frac{2R}{\pi}$	$-R^2\pi$	$-2R^2$
$\widehat{BD}$	2	$2R\pi$	$0$	$\frac{4R}{\pi}$	$0$	$8R^2$
$\widehat{DA}$	3	$R\pi$	$R$	$\frac{2R}{\pi}$	$R^2\pi$	$2R^2$
	$\Sigma$	$4R\pi$	/	/	$0$	$8R^2$

$$y_1 = -\frac{R \sin \frac{\pi}{2}}{\frac{\pi}{2}} = -\frac{2R}{\pi}$$

$$y_2 = \frac{2R \sin \frac{\pi}{2}}{\frac{\pi}{2}} = \frac{4R}{\pi}$$

$$y_3 = \frac{R \sin \frac{\pi}{2}}{\frac{\pi}{2}} = \frac{2R}{\pi}$$

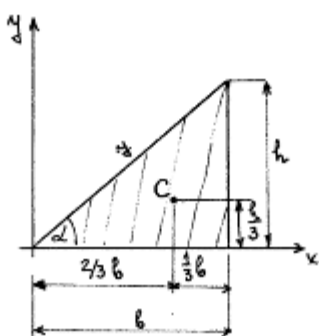
$$x_c = \frac{\sum_{i=1}^3 L_i x_i}{\sum L_i} = 0$$

$$y_c = \frac{\sum_{i=1}^3 L_i y_i}{\sum L_i} = \frac{8R^2}{4R\pi} = \frac{8 \cdot 4}{4 \cdot \pi} \Rightarrow \underline{y_c = 2,54 \text{ cm}}$$

$$C(0; 2,54) [\text{cm}]$$

**ZADATAK BR. 45**

Odrediti težište površine omeđene pravom koja prolazi kroz koordinatni početak.



$$y = kx ; k = \operatorname{tg} \alpha = \frac{h}{b}$$

$$y = f(x) = \frac{h}{b} x$$

$$A = \int_{x_1}^{x_2} f(x) dx = \int_0^b \frac{h}{b} x dx = \frac{h}{b} \int_0^b x dx = \frac{h}{b} \frac{x^2}{2} \Big|_0^b$$

$$A = \frac{h}{b} \frac{b^2}{2} \Rightarrow \underline{A = \frac{1}{2} hb}$$

$$\int_{x_1}^{x_2} x f(x) dx = \int_0^b x \frac{h}{b} x dx = \frac{h}{b} \int_0^b x^2 dx = \frac{h}{b} \frac{x^3}{3} \Big|_0^b = \frac{1}{3} b^2 h$$

$$x_c = \frac{\frac{1}{3} b^2 h}{\frac{1}{2} b h} \Rightarrow \underline{x_c = \frac{2}{3} b}$$

$$x_c = \frac{\int_{x_1}^{x_2} x f(x) dx}{\int_{x_1}^{x_2} f(x) dx}$$

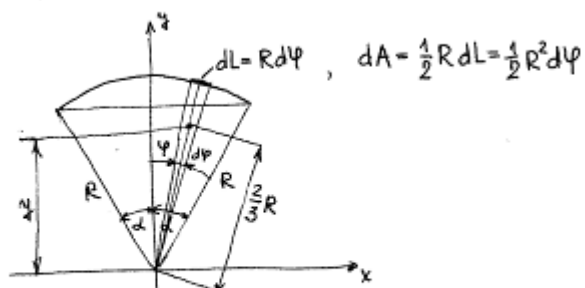
$$y_c = \frac{\frac{1}{2} \int_{x_1}^{x_2} [f(x)]^2 dx}{\int_{x_1}^{x_2} f(x) dx}$$

$$\frac{1}{2} \int_{x_1}^{x_2} [f(x)]^2 dx = \frac{1}{2} \int_0^b \left(\frac{h}{b} x\right)^2 dx = \frac{1}{2} \frac{h^2}{b^2} \int_0^b x^2 dx = \frac{1}{2} \frac{h^2}{b^2} \frac{x^3}{3} \Big|_0^b = \frac{1}{2} \frac{h^2}{b^2} \frac{b^3}{3} = \frac{1}{6} b h^2$$

$$y_c = \frac{\frac{1}{6} b h^2}{\frac{1}{2} b h} \Rightarrow \underline{y_c = \frac{1}{3} h}$$

**ZADATAK BR. 46**

Odrediti težište kružnog isečka.



$$y_c = \frac{\int y dA}{\int dA}$$

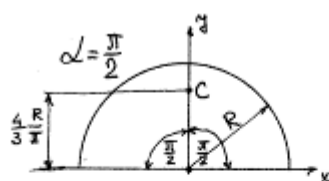
$$A = \int dA = \int_{-\alpha}^{\alpha} \frac{1}{2} R^2 d\psi = \frac{1}{2} R^2 \int_{-\alpha}^{\alpha} d\psi = \frac{1}{2} R^2 \psi \Big|_{-\alpha}^{\alpha} = \frac{1}{2} R^2 2\alpha ; \quad \underline{A = R^2 \alpha}$$

$$y = \frac{2}{3} R \cos \psi$$

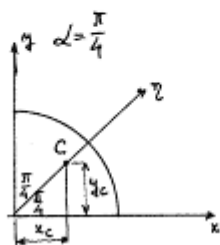
$$\int y dA = \int_{-\alpha}^{\alpha} \frac{2}{3} R \cos \psi \cdot \frac{1}{2} R^2 d\psi = \frac{1}{3} R^3 \int_{-\alpha}^{\alpha} \cos \psi d\psi = \frac{1}{3} R^3 \sin \psi \Big|_{-\alpha}^{\alpha} = \frac{1}{3} R^3 2 \sin \alpha = \frac{2}{3} R^3 \sin \alpha$$

$$y_c = \frac{\frac{2}{3} R^3 \sin \alpha}{R^2 \alpha}$$

$$\underline{y_c = \frac{2}{3} \frac{R \sin \alpha}{\alpha} ; \quad \alpha [\text{rad}]}$$



$$y_c = \frac{2}{3} \frac{R \sin \frac{\pi}{2}}{\frac{\pi}{2}} \Rightarrow \underline{y_c = \frac{4}{3} \frac{R}{\pi}}$$



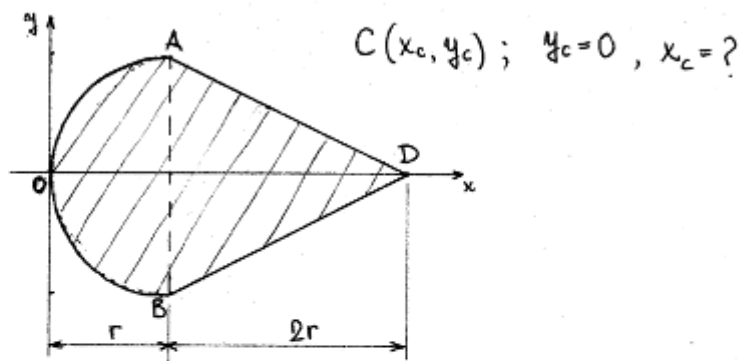
$$y_c = \frac{2}{3} \frac{R \sin \frac{\pi}{4}}{\frac{\pi}{4}} = \frac{2}{3} \frac{R \frac{\sqrt{2}}{2}}{\frac{\pi}{4}} = \frac{4\sqrt{2}}{3\pi} R$$

$$x_c = y_c = y_c \cos \frac{\pi}{4} = y_c \frac{\sqrt{2}}{2} = \frac{4\sqrt{2}}{3\pi} \frac{\sqrt{2}}{2} R$$

$$\underline{x_c = y_c = \frac{4}{3} \frac{R}{\pi}}$$

ZADATAK BR. 47

Odrediti položaj težišta C površine ograničene polkrugom AOB, poluprečnika R, i dvema dužima  $\overline{AD}$  i  $\overline{BD}$  jednakih dužina.



$$A = A_1 + A_2 = \frac{\pi r^2}{2} + \frac{1}{2} 2r \cdot 2r = \frac{\pi r^2}{2} + 2r^2 = \frac{r^2}{2} (\pi + 4)$$

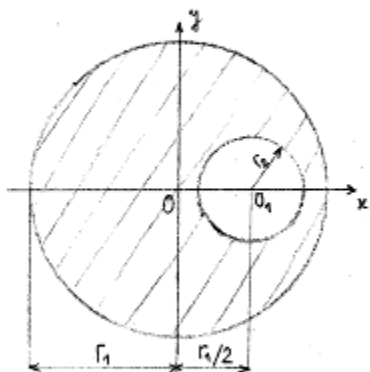
$$\begin{aligned} \sum A_i x_i &= \frac{\pi r^2}{2} \left( r - \frac{4r}{3\pi} \right) + 2r^2 \left( r + \frac{2r}{3} \right) = r^3 \left( \frac{\pi}{2} - \frac{2}{3} + 2 + \frac{4}{3} \right) = r^3 \left( \frac{\pi}{2} + \frac{8}{3} \right) \\ &= \frac{r^3}{6} (3\pi + 16) \end{aligned}$$

$$x_c = \frac{\sum A_i x_i}{A} = \frac{\frac{r^3}{6} (3\pi + 16)}{\frac{r^2}{2} (\pi + 4)} = \frac{r}{3} \cdot \frac{3\pi + 16}{\pi + 4}$$

$$\underline{x_c \approx 1,19 r}$$

**ZADATAK BR. 48**

Iz homogenog diska poluprečnika  $r_1$ , isečen je kružni otvor poluprečnika  $r_2$ . Središte  $O_1$  otvora nalazi se na rastojanju  $r_1/2$  od središta  $O$  diska. Odrediti položaj težišta diska sa tim kružnim otvorom.



$$y_c = 0$$

$$x_c = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2}$$

$$A_1 = r_1^2 \pi, \quad x_1 = 0; \quad A_1 x_1 = 0$$

$$A_2 = r_2^2 \pi, \quad x_2 = \frac{r_1}{2}; \quad A_2 x_2 = \frac{r_1 r_2^2 \pi}{2}$$

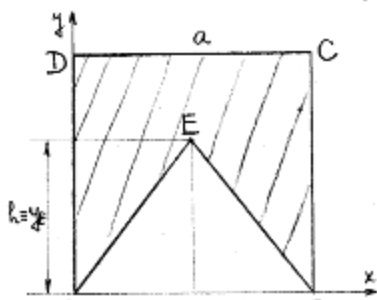
$$A_1 - A_2 = \pi (r_1^2 - r_2^2)$$

$$x_c = -\frac{r_1 r_2^2 \pi}{2 (r_1^2 - r_2^2) \pi} = -\frac{r_1 r_2^2}{2 (r_1^2 - r_2^2)}$$

$$\underline{\underline{C \left( -\frac{r_1 r_2^2}{2 (r_1^2 - r_2^2)}; 0 \right)}}$$

**ZADATAK BR. 49**

Iz homogene kvadratne ploče ABCD treba iseći jednakokraki trougao ABE, tako da tačka E bude težište preostale površine. Odrediti koordinate tačke E.



$$x_E = \frac{a}{2}$$

$$y_E = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2}$$

$$A_1 = a^2, \quad y_1 = \frac{a}{2}; \quad A_1 y_1 = \frac{a^3}{2}$$

$$A_2 = \frac{1}{2} a y_E; \quad y_2 = \frac{1}{3} y_E; \quad A_2 y_2 = \frac{a y_E^2}{6}$$

$$y_E = \frac{\frac{a^3}{2} - \frac{a y_E^2}{6}}{a^2 - \frac{1}{2} a y_E} = \frac{\frac{3a^3 - y_E^2}{6}}{\frac{2a - y_E}{2}} = \frac{3a - y_E^2}{3(2a - y_E)}$$

$$6a y_E - 3y_E^2 = 3a - y_E^2$$

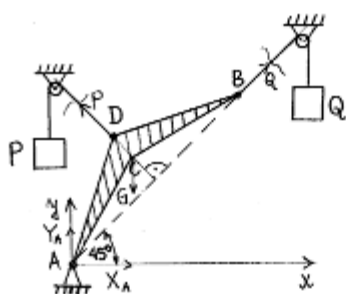
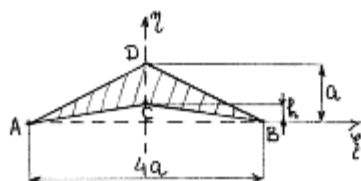
$$2y_E^2 - 6a y_E + 3a = 0$$

$$y_{E \pm} = \frac{6a \pm \sqrt{36a^2 - 24a^2}}{4} = \frac{6a \pm 2a\sqrt{3}}{4} = \frac{3}{2}a \pm \frac{1}{2}a\sqrt{3}$$

$$\underline{\underline{y_E = \frac{3}{2}a - \frac{1}{2}a\sqrt{3} = \frac{1}{2}a(3 - \sqrt{3})}}$$

**ZADATAK BR. 50**

Homogena pločica ACBD, težine  $G$ , sa težištem u tački  $C$ , dobijena je isecanjem trougaone pločice ACB iz trougaone pločice ABD. Teme  $A$  pločice vezano je zgloбно, a za teme  $B$  vezano je lako nerastegljivo uže o čiji drugi kraj visi teret  $Q = G/2$ . Uže je prebačeno preko malog kotura  $H$  tako da sa horizontalalom gradi ugao od  $45^\circ$  za ravnotežni položaj pločice koji je ostvaren pomoću tege  $P$ . Odrediti težinu tege  $P$ , koji je vezan za uže  $DE$  prebačeno preko malog kotura  $K$ , tako da uže ima pravac  $CD$ . Odrediti silu u zglobovima  $A$ .



$$C(x_c, y_c)$$

$$\sum \xi_c = 0$$

$$y_c = h = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2}$$

$$A_1 = \frac{1}{2} 4a \cdot a = 2a^2; \quad y_1 = \frac{1}{3} a$$

$$A_2 = \frac{1}{2} 4a \cdot h = 2ah; \quad y_2 = \frac{1}{3} h$$

$$h = \frac{2a^2 \cdot \frac{1}{3} a - 2ah \cdot \frac{1}{3} h}{2a^2 - 2ah}$$

$$2ha^2 - 2h^2 a = \frac{2}{3} a^3 - \frac{2}{3} h^2 a \quad / : a$$

$$2h^2 - \frac{2}{3} h^2 - 2ah + \frac{2}{3} a^2 = 0$$

$$\frac{4}{3} h^2 - 2ah + \frac{2}{3} a^2 = 0 \quad / \cdot 3$$

$$4h^2 - 6a \cdot h + 2a^2 = 0$$

$$h_{1/2} = \frac{6a \pm \sqrt{36a^2 - 32a^2}}{8} = \frac{6a \pm 2a}{8} = \frac{3a \pm a}{4}$$

$$h_1 = a, \quad h_2 = \frac{1}{2} a \Rightarrow h = \frac{1}{2} a$$

$$1) \sum X_i = 0: X_A - P \cos 45^\circ + Q \cos 45^\circ = 0 \Rightarrow X_A = (P - Q) \frac{\sqrt{2}}{2}$$

$$2) \sum Y_i = 0: Y_A + P \sin 45^\circ + Q \sin 45^\circ - G = 0 \Rightarrow Y_A = G - (P + Q) \frac{\sqrt{2}}{2}$$

$$3) \sum M_A = 0: P \cdot 2a - G \cdot x_G = 0$$

$$x_G = 2a \cdot \cos 45^\circ - \frac{1}{2} a \cos 45^\circ = 2a \frac{\sqrt{2}}{2} - \frac{1}{2} a \frac{\sqrt{2}}{2} = a \frac{\sqrt{2}}{2} (2 - \frac{1}{2}) = \frac{3\sqrt{2}}{4} a$$

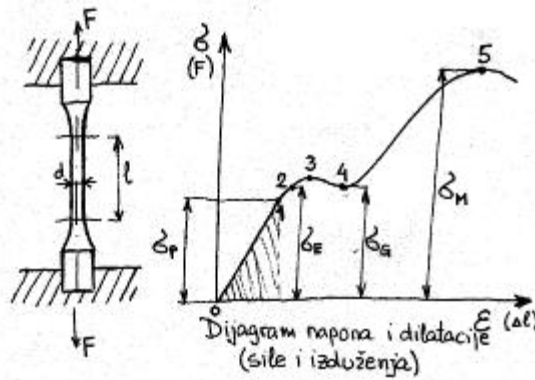
$$P \cdot 2a - G \frac{3\sqrt{2}}{4} a = 0 \Rightarrow P = G \frac{3\sqrt{2}}{8}$$

$$1) \Rightarrow X_A = (G \frac{3\sqrt{2}}{8} - G \frac{1}{2}) \frac{\sqrt{2}}{2} = G (\frac{3\sqrt{2}}{8} - \frac{1}{2}) \frac{\sqrt{2}}{2} = G \frac{6 - 4\sqrt{2}}{16}$$

$$2) \Rightarrow Y_A = G - G (\frac{3\sqrt{2}}{8} + \frac{1}{2}) \frac{\sqrt{2}}{2} = G \frac{10 - \sqrt{2}}{16}$$

# **OTPORNOST MATERIJALA**

1. AKSIJALNO NAPREZANJE



- 1 - granica proporcionalnosti
  - 2 - granica elastičnosti
  - 3 - početak razvlačenja
  - 4 - završetak (granica) razvlačenja
  - 5 - jačina (čvrstoća) materijala pri kidanju (istezanju)
- $\sigma_p$  [N/cm<sup>2</sup>] - napon na granici proporcionalnosti  
 $\sigma_E$  - napon na granici elastičnosti  
 $\sigma_M$  - čvrstoća materijala na kidanju

$\epsilon = \frac{\Delta l}{l}$  ;  $\Delta l$  - izduženje (skraćenje)

dilatacija (vzdužna) - relativno izduženje (skraćenje)

$\Delta l = l_1 - l$  ;  $l_1$  - dužina posle deformacije  
 $l$  - prvobitna dužina

$\sigma = \frac{F}{A}$  [  $\frac{N}{cm^2}$  ] - napon

$\sigma = E \cdot \epsilon$  - I oblik Hukovog zakona (zakon proporcionalnosti)

Napon je linearna f-ja dilatacije u oblasti proporcionalnosti za slučaj aksijalnog naprezanja

$\sigma = \frac{F}{A} = E \frac{\Delta l}{l} \Rightarrow \Delta l = \frac{\sigma \cdot l}{E} = \frac{F \cdot l}{E \cdot A} = \frac{F \cdot l}{U}$  - II oblik Hukovog zakona

$E$  [N/cm<sup>2</sup>] - koeficijent proporcionalnosti (modul elastičnosti), Jungov modul

$U$  [N] - krutost štapa (na istezanje)

$l_1 = l + \Delta l = l + l \epsilon = l(1 + \epsilon)$

$\epsilon_p = -\mu \epsilon$  - poprečna dilatacija (kontrakcija) štapa

$\mu$  - Poasonov koeficijent proporcionalnosti

$\epsilon_p = \frac{\Delta A}{A} = \frac{A_1 - A}{A} = \frac{A_1}{A} - 1$

$\epsilon_v = \frac{\Delta V}{V}$  - kubna (zapreminska) dilatacija

$\epsilon_v = (1 - 2\mu) \epsilon$

$\epsilon_t = \alpha (t_2 - t_1) = \alpha \Delta t$  - dilatacija usled promene temperature (temperaturna dilatacija)

$\sigma_t = E \cdot \epsilon_t = E \alpha (t_2 - t_1)$

$\Delta l_t = \alpha (t_2 - t_1) \cdot l$  ;  $\alpha$  - termički koeficijent linearnog širenja ;  
 Odgovara izduženju štapa dužine 1 cm pri povećanju temperature za 1°C.

**ZADATAK BR. 1**

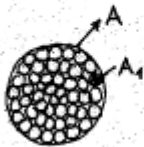
Čelična probna epruveta prečnika  $d = 20 \text{ mm}$  i dužine  $l = 10d$  iskidana je na mašini za kidanje pri najvećem opterećenju od  $F_M = 142,1 \text{ kN}$ . Njena dužina posle kidanja iznosi  $l_1 = 247 \text{ mm}$ . Kolika je čvrstoća materijala na kidanje i koliko je izduženje?

$$\sigma_M = \frac{F_M}{A} = \frac{F_M}{\frac{d^2 \pi}{4}} = \frac{142,1}{\frac{2^2 \pi}{4}} = 45,25 \frac{\text{kN}}{\text{cm}^2}$$

$$\Delta l = l_1 - l = l_1 - 10d = 247 - 200 = 47 \text{ mm}$$

**ZADATAK BR. 2**

Čelično uže dizalice treba da nosi teret od  $F = 100 \text{ kN}$ . Koliko žica prečnika  $d = 1,13 \text{ mm}$  treba da ima uže ako je dozvoljeni napon na istezanje  $\sigma_{de} = 25 \text{ kN/cm}^2$ ?



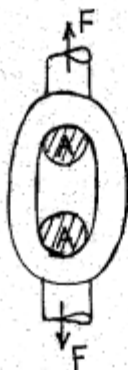
$$\sigma_{de} \geq \sigma = \frac{F}{A} \Rightarrow A \geq \frac{F}{\sigma_{de}} = \frac{100}{25} = 4 \text{ cm}^2$$

$$A = n A_1, \quad A_1 = \frac{d^2 \pi}{4} = 1 \text{ mm}^2 = 10^{-2} \text{ cm}^2$$

$$n = \frac{A}{A_1} = \frac{4}{10^{-2}} = 400 \text{ komada žica}$$

**ZADATAK BR. 3**

Karika lanca dizalice prenosi silu  $F = 31,4 \text{ kN}$ . Odrediti prečnik kružnog poprečnog preseka karike ako je dozvoljeni napon na istezanje  $\sigma_{de} = 20 \text{ kN/cm}^2$ .



$$\sigma_{de} \geq \sigma = \frac{F}{2A} \Rightarrow A \geq \frac{F}{2\sigma_{de}}$$

$$\frac{d^2 \pi}{4} \geq \frac{F}{2\sigma_{de}} \Rightarrow d \geq \sqrt{\frac{2F}{\sigma_{de} \pi}}$$

$$d \geq 1 \text{ cm}$$

**ZADATAK BR. 4**

Bakarna žica dužine  $l = 1,5 \text{ m}$  i površine poprečnog preseka  $3 \text{ mm}^2$  izduži se pod dejstvom sile  $F = 20 \text{ N}$  za  $\Delta l = 0,9 \text{ mm}$ .

Odrediti modul elastičnosti.

$$\Delta l = \frac{F \cdot l}{E \cdot A} \Rightarrow E = \frac{F \cdot l}{A \cdot \Delta l} = \frac{20 \cdot 1,5 \cdot 10^2}{0,9 \cdot 10^{-1} \cdot 3 \cdot 10^{-2}} = 1,11 \cdot 10^6 \frac{\text{N}}{\text{cm}^2}$$

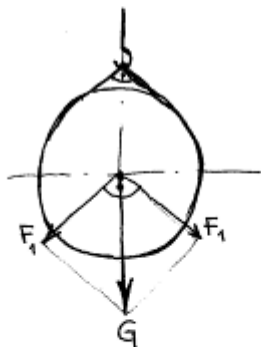
**ZADATAK BR. 5**

Kotao težine  $29 \text{ kN}$  obešen je pomoću lanca ~~na~~ kuku. Krajevi lanca čine ugao  $90^\circ$ .

a) Odrediti sile u delovima lanca;

b) Dimenzionisati kariku lanca kružnog preseka ako je  $\sigma_{de} = 6000 \frac{\text{N}}{\text{cm}^2}$ .

$$G = 29 \text{ kN} ; \sigma_{de} = 6000 \frac{\text{N}}{\text{cm}^2} ; F_1, d = ?$$



$$a) G = F_1 \sqrt{2}$$

$$F_1 = \frac{G}{\sqrt{2}} = 20506 \text{ N}$$

$$b) \sigma_{de} \geq \frac{F_1}{2A}$$

$$A = \frac{d^2 \pi}{4} = \frac{F_1}{2\sigma_{de}}$$

$$d = \sqrt{\frac{2F_1}{\pi \sigma_{de}}} \approx 1,5 \text{ cm}$$

**ZADATAK BR. 6**

Horizontalni čelični štap, dužine  $4 \text{ m}$ , kvadratnog poprečnog preseka  $1 \text{ cm}^2$ , izdužio se pri zagrevanju za  $1 \text{ mm}$ .

Kolikim silama treba pritiskati na krajevima štapa da bi se izduženje poništilo?  $E = 2 \cdot 10^7 \text{ N/cm}^2$

$$l = 4 \text{ m}, A = 1 \text{ cm}^2, \Delta l = 1 \text{ mm} ; F = ?$$

$$F = A \cdot \sigma = A \cdot E \epsilon = A \cdot E \cdot \frac{\Delta l}{l} = 1 \cdot 2 \cdot 10^7 \cdot \frac{0,1}{400} = 5000 \text{ N}$$

**ZADATAK BR. 7**

Čelični štap AB obostrano je uklešten pri temperaturi  $0^{\circ}\text{C}$ .  
 Odrediti napon u štapu ako je temperatura porasla na  $40^{\circ}\text{C}$ .

$$\alpha = 0,0000125 ; E = 2 \cdot 10^7 \text{ N/cm}^2$$

$$t_1 = 0^{\circ}\text{C} , t_2 = 40^{\circ}\text{C} \Rightarrow \Delta t = 40^{\circ}\text{C}$$

$$\sigma = E \cdot \varepsilon_t = E \cdot \alpha \Delta t = 2 \cdot 10^7 \cdot 0,0000125 \cdot 40 = 10000 \text{ N/cm}^2$$

**ZADATAK BR. 8**

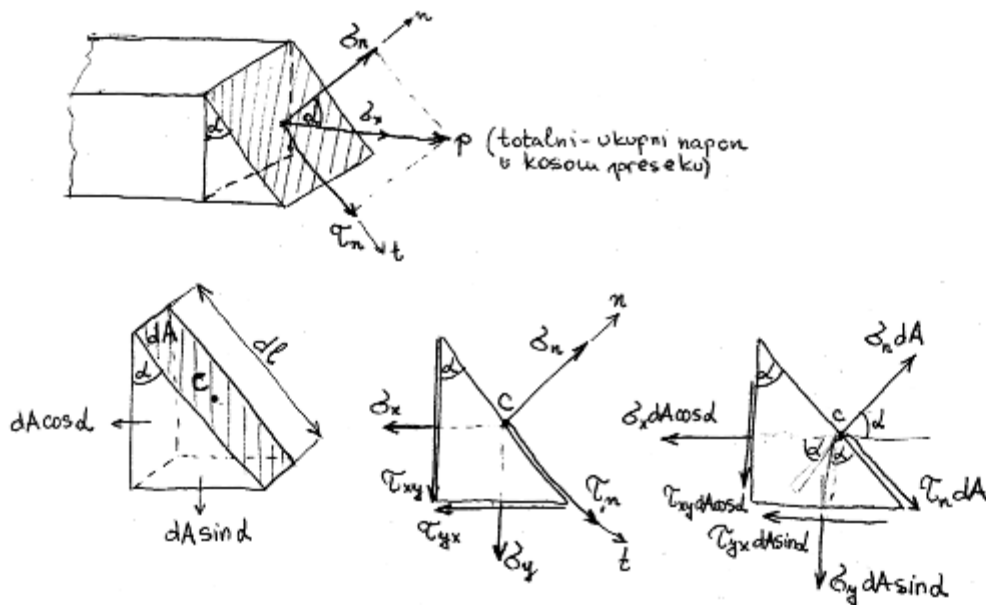
Na kom razmaku treba namestiti, pri temperaturi  $t_1 = 10^{\circ}\text{C}$ ,  
 krajeve šina, dužine 12 m, da bi se pri temperaturi  $t_2 = 50^{\circ}\text{C}$   
 dodirivale ne izazivajući napon? Koliki se napon javlja u  
 šini ako temperatura poraste do  $t_3 = 70^{\circ}\text{C}$ ?

$$\alpha = 0,0000125 ; E = 2 \cdot 10^7 \text{ N/cm}^2$$

$$\varepsilon_t = \alpha (t_2 - t_1) = \frac{\Delta l}{l} \Rightarrow \Delta l = l \cdot \alpha (t_2 - t_1) = 0,6 \text{ cm} = 6 \text{ mm}$$

$$\sigma = E \cdot \varepsilon'_t = E \cdot \alpha (t_3 - t_2) = 5000 \text{ N/cm}^2$$

## 1.2 Naponsko stanje aksijalno napregnutog štapa



$$\begin{aligned}
 1. \sum N = 0 &: \sigma_n dA - \sigma_x dA \cos^2 \alpha - \sigma_y dA \sin^2 \alpha - \tau_{xy} dA \cos \alpha \sin \alpha - \tau_{yx} dA \sin \alpha \cos \alpha = 0 \\
 2. \sum T = 0 &: \tau_n dA - \sigma_x dA \cos \alpha \sin \alpha + \sigma_y dA \sin \alpha \cos \alpha + \tau_{xy} dA \cos^2 \alpha - \tau_{yx} dA \sin^2 \alpha = 0 \\
 3. \sum M_c = 0 &: \tau_{xy} dA \cos \alpha \cdot \frac{dl}{2} \sin \alpha - \tau_{yx} dA \sin \alpha \cdot \frac{dl}{2} \cos \alpha = 0
 \end{aligned}$$

$$3. \Rightarrow \tau_{xy} = \tau_{yx}$$

$$1. \Rightarrow \sigma_n = \sigma_x \cos^2 \alpha + \sigma_y \sin^2 \alpha + 2\tau_{xy} \cos \alpha \sin \alpha$$

$$2. \Rightarrow \tau_n = (\sigma_x - \sigma_y) \sin \alpha \cos \alpha - \tau_{xy} (\cos^2 \alpha - \sin^2 \alpha)$$

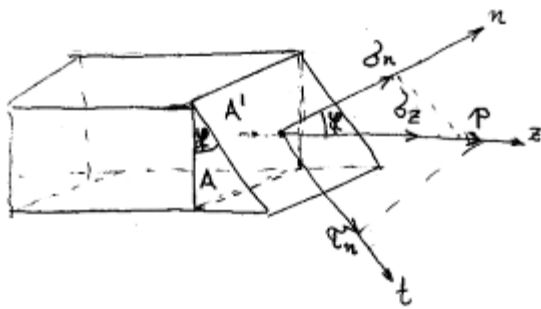
$$\left. \begin{aligned}
 \cos^2 \alpha &= \frac{1}{2} (\cos 2\alpha + 1) ; \quad \sin^2 \alpha = \frac{1}{2} (1 - \cos 2\alpha) ; \\
 2 \sin \alpha \cos \alpha &= \sin 2\alpha ; \quad \cos^2 \alpha - \sin^2 \alpha = \cos 2\alpha
 \end{aligned} \right\} \Rightarrow$$

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\alpha + \tau_{xy} \sin 2\alpha$$

$$\tau_n = \frac{\sigma_x - \sigma_y}{2} \sin 2\alpha - \tau_{xy} \cos 2\alpha$$

**ZADATAK BR. 9**

Normalni napon po poprečnom preseku zategnutog štapa iznosi  $600 \text{ N/cm}^2$ . Za neki kosi presek je tangencijalni napon  $260 \text{ N/cm}^2$ . Odrediti položaj kosog preseka i izračunati brojnu vrednost normalnog napona u tom preseku.



$$p = \frac{F}{A'} = \frac{F}{A} \cos \varphi = \sigma_z \cos \varphi$$

$$A' = \frac{A}{\cos \varphi}$$

$$\sigma_n = p \cos \varphi = \sigma_z \cos^2 \varphi$$

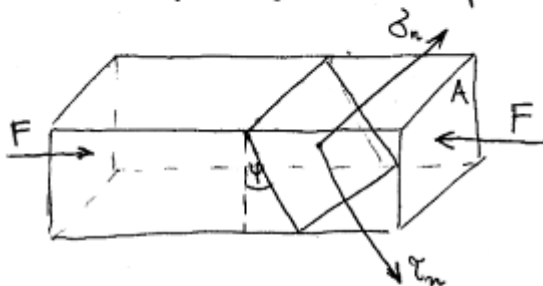
$$\tau_n = p \sin \varphi = \sigma_z \sin \varphi \cos \varphi = \frac{1}{2} \sigma_z \sin 2\varphi$$

$$\sin 2\varphi = \frac{2\tau_n}{\sigma_z} = \frac{2 \cdot 260}{600} = 0,866 \Rightarrow \varphi = 30^\circ$$

$$\sigma_n = 600 \cdot \cos^2 30^\circ = 600 \left(\frac{\sqrt{3}}{2}\right)^2 = 450 \text{ N/cm}^2$$

**ZADATAK BR. 10**

Betonski stub poprečnog preseka  $30 \times 30 \text{ cm}$  pritisnut je silom  $F$ . Odrediti veličinu ove sile ako je normalan napon u kosom preseku, pod uglom  $30^\circ$  prema poprečnom preseku,  $10 \text{ N/cm}^2$ . Koliki je tangencijalni napon za taj presek?



$$\varphi = 30^\circ, \sigma_n = 10 \text{ N/cm}^2$$

$$A = 30 \times 30 \text{ [cm]}$$

$$F = ? \quad \tau_n = ?$$

$$p = \frac{F}{A}$$

$$\sigma_n = p \cos \varphi = \sigma_z \cos^2 \varphi = \frac{F}{A} \cos^2 \varphi$$

$$\sigma_z = \frac{\sigma_n}{\cos^2 \varphi} = \frac{40}{3} \frac{\text{N}}{\text{cm}^2}$$

$$F = A \sigma_z = 30 \cdot 30 \cdot \frac{40}{30} \Rightarrow F = 12000 \text{ N}$$

$$\tau_n = \frac{1}{2} \sigma_z \sin 2\varphi = 5,77 \frac{\text{N}}{\text{cm}^2}$$

**ZADATAK BR. 11**

Stub kružnog poprečnog preseka izložen je dejstvu sile 20 kN.  
Dimenzionisati stub pod uslovom da najveći tangencijalni napon ne pređe vrednost  $\tau_{00} \text{ N/cm}^2$ .

$$\tau_{\max} = \frac{1}{2} \sigma_z \sin 2\varphi = \frac{\sigma_z}{2} \Rightarrow \sin 2\varphi = 1 \Rightarrow \varphi = 45^\circ$$

$$\Rightarrow \sigma_z = 2\tau_{\max} = 1400 \text{ N/cm}^2$$

$$\sigma_z = \frac{F}{A} \Rightarrow A = \frac{d^2 \pi}{4} = \frac{F}{\sigma_z}$$

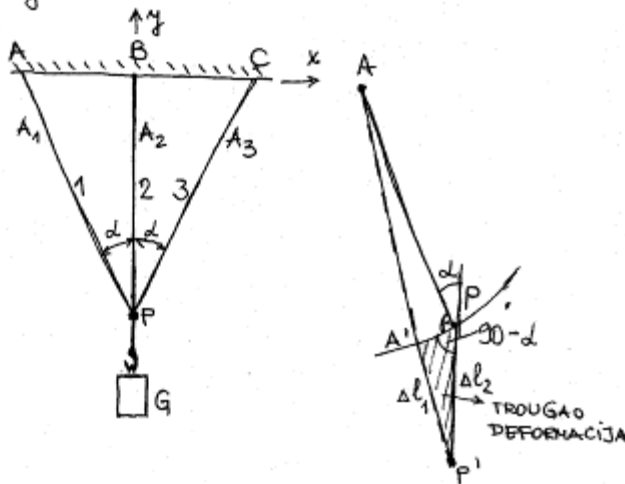
$$d = 2 \sqrt{\frac{F}{\pi \sigma_z}} = 4,26 \text{ cm} ; \text{ usvaja se } d = 45 \text{ mm}$$

## 1.3 Aksijalno naprezanje - statički neodređeni slučajevi

## ZADATAK BR. 12

Tri štapa od istog materijala, dužine  $l_1 = l_3$ ,  $l_2$ , poprečnih preseka  $A_1 = A_3$  i  $A_2$  vezana su zglobno u A, B i C i skupno u tački P o koju je obešen teret G.

Odrediti sile u štapovima ako krajnji štapani grade sa srednjim ugao  $\alpha$ .



$$1. \sum X_i = -F_1 \sin \alpha + F_3 \sin \alpha = 0 ; F_1 = F_3$$

$$2. \sum Y_i = 2F_1 \cos \alpha + F_2 - G = 0$$

$$\overline{AP'} = \overline{CP'} = l'_1 = l_1 (1 + \epsilon_1)$$

$$\overline{PP'} = \Delta l_2, \overline{A'P'} = \Delta l_1 ; \overline{PA'} \perp \overline{AP} (\overline{AA'} = l_1) \Rightarrow \Delta l_1 = \Delta l_2 \sin(90^\circ - \alpha) = \Delta l_2 \cos \alpha$$

$$3. \Delta l_1 = \Delta l_2 \cos \alpha$$

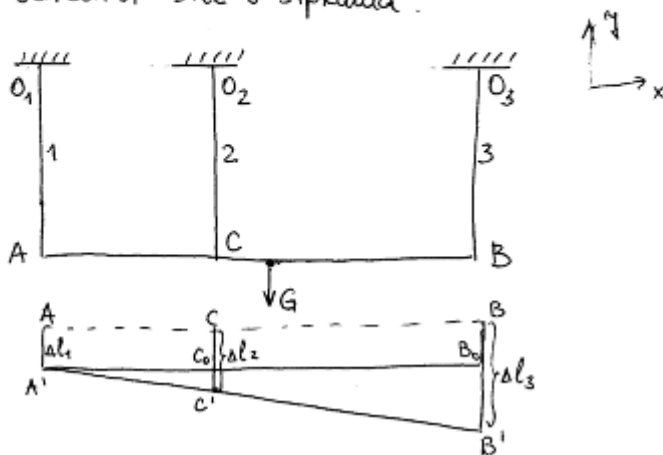
$$\frac{F_1 l_1}{EA_1} = \frac{F_2 l_2}{EA_2} \cos \alpha = \frac{F_2 l_1}{EA_2} \cos^2 \alpha$$

$$F_1 = \frac{A_1}{A_2} F_2 \cos^2 \alpha = (k \cos^2 \alpha) F_2 = \frac{G_3 F_2}{2 \cos \alpha} ; k = \frac{A_1}{A_2}$$

$$F_1 = \frac{k \cos^2 \alpha}{1 + 2k \cos^3 \alpha} G ; F_2 = \frac{1}{1 + 2k \cos^3 \alpha} G$$

**ZADATAK BR. 13**

Greda AB, težine  $G = 28 \text{ N}$ , dužine  $\overline{AB} = l = 60 \text{ cm}$  obešena je pomoću tri jednake čelične šipke o tavan,  $\overline{AC} = \overline{BC}/2$ .  
 Odrediti sile u šipkama.



$$1. \sum Y_i = F_1 + F_2 + F_3 - G = 0$$

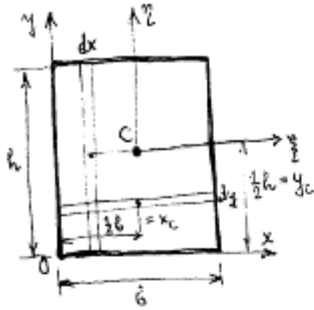
$$2. \sum M_B = 60 F_1 + 40 F_2 - 30 G = 0$$

$$(\Delta l_2 - \Delta l_1) \cdot l/3 = (\Delta l_3 - \Delta l_1) : l \Rightarrow$$

$$\Rightarrow 3. -2\Delta l_1 + 3\Delta l_2 - \Delta l_3 = 0 \quad ; \quad \Delta l_i = \frac{F_i \cdot l_i}{E_i \cdot A_i}$$

$$\Rightarrow \left. \begin{array}{l} 1. F_1 + F_2 + F_3 = G \\ 2. 6F_1 + 4F_2 = 3G \\ 3. -2F_1 + 3F_2 - F_3 = 0 \end{array} \right\} \Rightarrow \begin{array}{l} F_1 = \frac{2G}{7} = 8 \text{ N} \\ F_2 = \frac{9G}{28} = 9 \text{ N} \\ F_3 = \frac{11G}{28} = 11 \text{ N} \end{array}$$

**MOMENTI INERCIJE RAVNIH PRESEKA**



$$A = b \cdot h$$

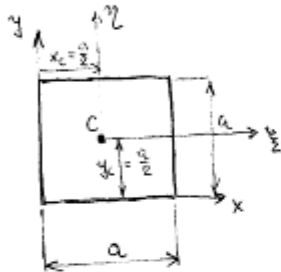
$$I_x = \iint_A y^2 dA = \int_0^h y^2 b \cdot dy = b \int_0^h y^2 dy = \frac{b}{3} y^3 \Big|_0^h = \frac{bh^3}{3}$$

$$I_y = \iint_A x^2 dA = \int_0^b x^2 h \cdot dx = h \int_0^b x^2 dx = \frac{h}{3} x^3 \Big|_0^b = \frac{hb^3}{3}$$

$$I_o = I_x + I_y = \frac{bh}{3} (b^2 + h^2)$$

$$I_{x_c} = I_x - A \cdot y_c^2 = \frac{bh^3}{3} - bh \cdot \left(\frac{h}{2}\right)^2 = \frac{bh^3}{12}$$

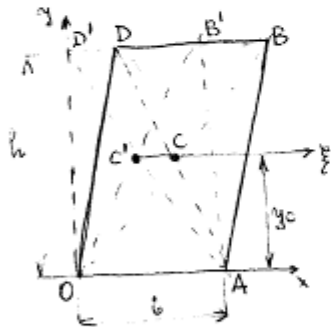
$$I_{y_c} = I_y - A \cdot x_c^2 = \frac{hb^3}{3} - bh \cdot \left(\frac{b}{2}\right)^2 = \frac{hb^3}{12}$$



$$A = a^2$$

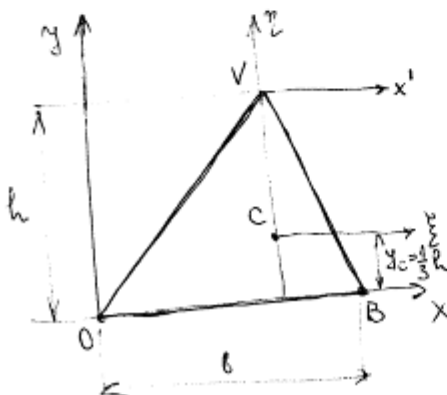
$$I_x = I_y = \frac{a^4}{3}$$

$$I_{x_c} = I_{y_c} = \frac{a^4}{12}$$



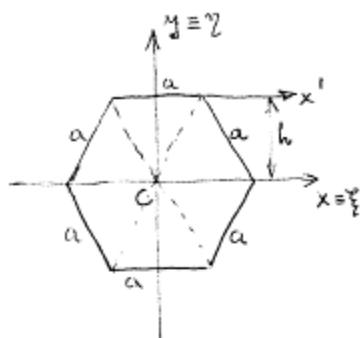
$$I_x = \frac{bh^3}{3}$$

$$I_{x_c} = \frac{bh^3}{12}$$

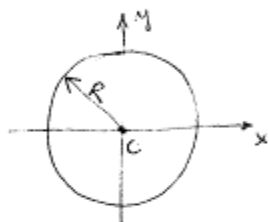


$$I_x = \frac{bh^3}{12} \quad ; \quad I_{x_c} = I_x + A y_c^2 = \frac{bh^3}{12} + \left(\frac{1}{2}bh\right) \left(\frac{2}{3}h\right)^2 = \frac{bh^3}{4}$$

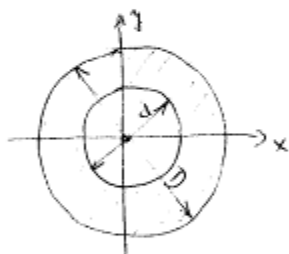
$$I_{y_c} = \frac{bh^3}{36}$$



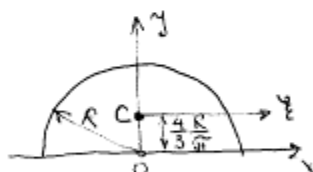
$$I_x = I_\xi = 2 \left[ 2 \cdot \frac{bh^3}{12} + 1 \cdot \frac{bh^3}{4} \right] = \frac{5}{3} bh^3 ; b = a, h = a \frac{\sqrt{3}}{2}$$



$$I_x = I_y = \frac{R^4 \pi}{4} = \frac{d^4 \pi}{64} ; d = 2R$$

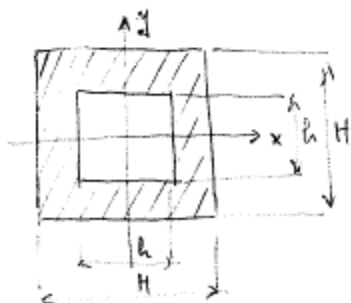


$$I_x = I_y = \frac{(D^4 - d^4) \pi}{64} = \frac{D^4 \pi}{64} (1 - \psi^4) ; \psi = \frac{d}{D} = \frac{r}{R}$$

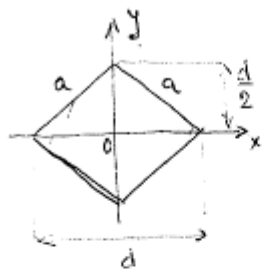


$$I_x = I_y = \frac{R^4 \pi}{8}$$

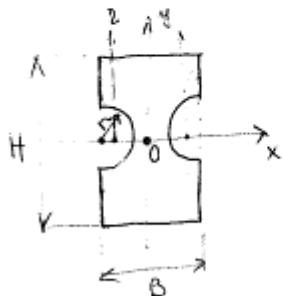
$$I_\xi = I_x - Ay_c^2 = \frac{R^4 \pi}{8} - \left( \frac{1}{2} R^2 \pi \right) \left( \frac{4}{3} \frac{R}{\pi} \right)^2 = \frac{9\pi^2 - 64}{72 \cdot \pi} R^4$$



$$I_x = I_y = \frac{H^4 - h^4}{12}$$

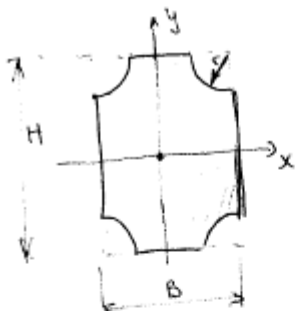


$$I_x = I_y = 2 \frac{d \cdot \left(\frac{d}{2}\right)^3}{12} = 2 \frac{(a\sqrt{2}) \left(\frac{a\sqrt{2}}{2}\right)^3}{12} = \frac{a^4}{12}$$



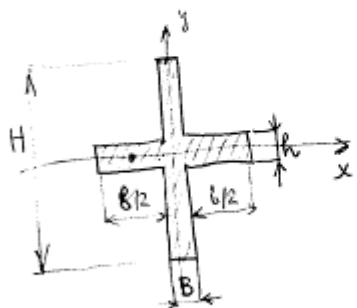
$$I_x = \frac{BH^3}{12} - \frac{r^4\pi}{4}$$

$$I_y = \frac{HB^3}{12} - 2 \left[ \underbrace{\frac{r^4\pi}{8} - \frac{r^2\pi}{2} \left(\frac{4r}{3\pi}\right)^2}_{I_y} + \frac{r^2\pi}{2} \left(\frac{B}{2} - \frac{4r}{3\pi}\right)^2 \right]$$



$$I_x = \frac{BH^3}{12} - 2 \left[ \frac{r^4\pi}{8} - \left(\frac{1}{2}r^2\pi\right) \left(\frac{4r}{3\pi}\right)^2 + \frac{r^2\pi}{2} \left(\frac{H}{2} - \frac{4r}{3\pi}\right)^2 \right]$$

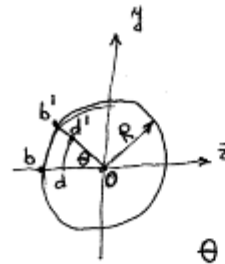
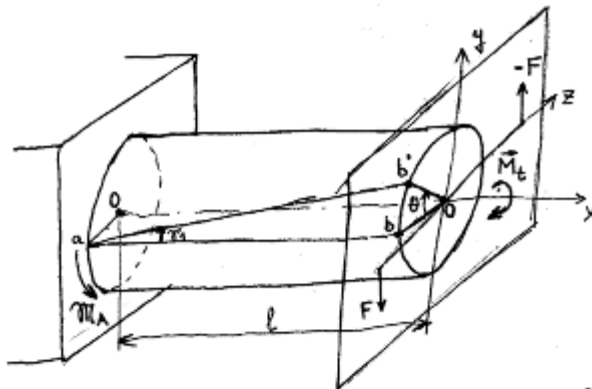
$$I_y = \frac{B^3H}{12} - 2 \left[ \frac{r^4\pi}{8} - \left(\frac{1}{2}r^2\pi\right) \left(\frac{4r}{3\pi}\right)^2 + \frac{r^2\pi}{2} \left(\frac{B}{2} - \frac{4r}{3\pi}\right)^2 \right]$$



$$I_x = \frac{BH^3}{12} + \frac{bh^3}{12}$$

$$I_y = \frac{HB^3}{12} + 2 \left[ \frac{h \left(\frac{B}{2}\right)^3}{12} + \left(\frac{b}{2} \cdot h\right) \left(\frac{B}{2} + \frac{b}{4}\right)^2 \right]$$

**2. UVIJANJE (TORZIJA)**



$\theta$  - ugao uvijanja

Kod svicanja :  $\tau = G \cdot \gamma$  ;  $\tau$  [N/cm<sup>2</sup>] - tangencijalni napon  
 $G$  [N/cm<sup>2</sup>] - modul klizanja  
 $\gamma$  - deformacija (klizanje)

$\tau = \frac{M_t \cdot r}{I_o}$  ;  $M_t$  [Ncm] - moment uvijanja (torzioni moment)

$\tau_{max} = \frac{M_t \cdot R}{I_o} = \frac{M_t}{W_o}$

I glavna j-na uvijanja

$I_o$  - polarni moment inercije poprečnog preseka za tačku O - središte kružnog preseka [cm<sup>4</sup>]

$W_o$  - polarni otporni moment poprečnog preseka za osu O [cm<sup>3</sup>]

$W_o = \frac{I_o}{R}$

$\theta$  [rad] =  $\frac{l \gamma}{R} = \frac{l}{R} \frac{\tau_{max}}{G} = \frac{M_t l}{G I_o}$   
 $\theta$  [°] =  $\frac{180^\circ}{\pi} \theta = \frac{180}{\pi} \frac{M_t l}{G I_o}$

II glavna j-na uvijanja

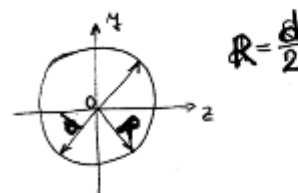
\* Kružni poprečni presek :

$I_o = \frac{1}{2} R^4 \pi = \frac{1}{32} d^4 \pi \approx \frac{1}{10} d^4$

$\Rightarrow W_o = \frac{1}{2} R^3 \pi = \frac{1}{16} d^3 \pi \approx \frac{1}{5} d^3$

$\Rightarrow \tau_{max} = \frac{2 M_t}{R^3 \pi} = \frac{16 M_t}{d^3 \pi} \approx \frac{5 M_t}{d^3}$

$\tau_{max} \leq \tau_{dt} \Rightarrow \left\{ d \geq \sqrt[3]{\frac{16 M_t}{\pi \tau_{dt}}} \approx \sqrt[3]{\frac{5 M_t}{\tau_{dt}}} \right.$  [cm]  
 [Ncm]  
 [Ncm<sup>2</sup>]



**ZADATAK BR. 14**

Čelična konzola, kružnog poprečnog preseka, opterećena je na slobodnom kraju momentom uvijanja  $M_t = 800$  [Nm].

Dimenzionisati konzolu ako je dozvoljeni napon na uvijanje  $200$  N/cm<sup>2</sup>.

Odrediti ugao uvijanja za raspon konzole  $l = 2$  m.  $G = 8 \cdot 10^5$  N/cm<sup>2</sup>.

$$\tau_{\max} = \frac{M_t \cdot R}{I_o} = \frac{M_t}{W_o} \leq \tau_{dt} \Rightarrow W_o \geq \frac{M_t}{\tau_{dt}}$$

$$W_o = \frac{I_o}{R} = \frac{1}{2} R^3 \pi = \frac{1}{16} d^3 \pi \geq \frac{M_t}{\tau_{dt}} \Rightarrow d \geq \sqrt[3]{\frac{16 M_t}{\pi \tau_{dt}}} = 12,68 \text{ cm}$$

usvaja se  $d = 13$  cm

$$\tau_{\max} = \frac{M_t}{W_o} = \frac{16 M_t}{d^3 \pi} = 185,45 \frac{\text{N}}{\text{cm}^2} < \tau_{dt} = 200 \frac{\text{N}}{\text{cm}^2}$$

$$\theta = \frac{M_t \cdot l}{G I_o} = \frac{l}{d} \frac{\tau_{\max}}{G} = 0,00713692 \text{ [rad]}$$

$$\theta^\circ = \frac{180^\circ}{\pi} \theta = 0,41^\circ = 0^\circ 24' 31''$$

**ZADATAK BR. 15**

Koliki je najveći tangencijalni napon grede kružnog poprečnog preseka, prečnika 110 mm, ako je izložena momentu uvijanja  $M_t = 1,4$  kNm?

$$\tau_{\max} = \frac{M_t}{W_o} = \frac{M_t}{\frac{1}{16} d^3 \pi} = \frac{16 M_t}{d^3 \pi} = \frac{16 \cdot 1,4 \cdot 10^3 \cdot 10^2}{11^3 \cdot \pi} \approx 535,7 \frac{\text{N}}{\text{cm}^2}$$

**ZADATAK BR. 16**

Greda kružnog poprečnog preseka, prečnika 100 mm, ima tangencijalni napon  $350$  N/cm<sup>2</sup> u tački poprečnog preseka na rastojanju 10 mm od središta.

Koliki je moment uvijanja grede?  $[d = 10 \text{ cm}, R = \frac{d}{2} = 5 \text{ cm}]$

$$\tau_{\max} = \frac{2 M_t}{R^3 \pi} \Rightarrow M_t = \frac{1}{2} \tau_{\max} R^3 \pi$$

$$\tau = \frac{M_t r}{I_o}, \tau_{\max} = \frac{M_t R}{I_o} \Rightarrow \frac{\tau}{\tau_{\max}} = \frac{r}{R} \Rightarrow \tau_{\max} = 1750 \frac{\text{N}}{\text{cm}^2}$$

$$\Rightarrow M_t = 3,5 \text{ kNm}$$

**ZADATAK BR. 17**

Vratilo kružnog poprečnog preseka, prečnika 80 mm, prenosi snagu 300 Nm/s pri 240 obr./min.

Odrediti najveći tangencijalni napon i ugao uvijanja vratila ako je njegova dužina 5 m, a modul klizanja 800 kN/cm<sup>2</sup>.

$$P = 300 \left[ \frac{\text{Nm}}{\text{s}} \right], \quad n = 240 \left[ \frac{\text{ob}}{\text{min}} \right], \quad d = 80 \text{ mm} = 8 \text{ cm}$$

$$l = 5 \text{ [m]}, \quad G = 800 \text{ [kN/cm}^2\text{]}$$

$$P = M_t \cdot \omega = M_t \cdot \frac{\pi n}{30} \Rightarrow M_t = \frac{30 P}{\pi n} = \frac{30 \cdot 300}{\pi \cdot 240} = 12 \text{ Nm}$$

$$\tau_{\max} = \frac{16 M_t}{d^3 \pi} = \frac{16 \cdot 1200}{8^3 \pi} = 11,94 \frac{\text{N}}{\text{cm}^2}$$

$$\theta = \frac{M_t l}{G I_0} = \frac{M_t l}{G \frac{\pi d^4}{32}} = \frac{32 M_t l}{G d^4 \pi} = \frac{32 \cdot 12 \cdot 10^2 \cdot 500}{800 \cdot 10^3 \cdot 8^4 \pi} = 0,001865 \text{ rad}$$

**ZADATAK BR. 18**

Vratilo dužine 6 m, kružnog poprečnog preseka, R = 100 mm, uvija se za 4°. Koliki je najveći tangencijalni napon ako je materijal čelik (G = 8 · 10<sup>5</sup> N/cm<sup>2</sup>) ?

$$l = 6 \text{ m} = 600 \text{ cm}$$

$$R = 100 \text{ mm} = 10 \text{ cm} \Rightarrow d = 5 \text{ cm}$$

$$\theta = 4^\circ$$

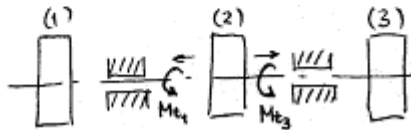
$$\tau_{\max} = ?$$

$$\theta = \frac{180}{\pi} \frac{l}{d} \frac{\tau_{\max}}{G} \Rightarrow \tau_{\max} = \frac{\theta \pi d G}{180 \cdot l} = \frac{4 \cdot \pi \cdot 5 \cdot 8 \cdot 10^5}{180 \cdot 600} = 465,42 \frac{\text{N}}{\text{cm}^2}$$

**ZADATAK BR. 19**

Transmisiono vratilo prima preko kaišnika (2) snagu 7500 W, a odaje preko kaišnika (1) snagu  $P_1 = 3000$  W i preko kaišnika (3) snagu  $P_3 = 4500$  W.

Dimenzionisati vratilo kružnog poprečnog preseka ako je  $\tau_{dt} = 120$  N/cm<sup>2</sup> i  $n = 175$  %min.



$$M_{t_i} = \frac{P_i}{\omega} = \frac{P}{\frac{\pi n}{30}} = \frac{30 P_i}{\pi \cdot n}$$

$$M_{t_1} = \frac{30 P_1}{\pi n} = \frac{30 \cdot 3000}{\pi \cdot 175} = 164 \text{ Nm} = 16400 \text{ Ncm}$$

$$M_{t_2} = 41000 \text{ Ncm}$$

$$M_{t_3} = \frac{30 P_3}{\pi \cdot n} = M_{t_2} - M_{t_1} = 24600 \text{ Ncm}$$

$$d \geq \sqrt[3]{\frac{5 M_{t_{\max, 1,3}}}{\tau_{dt}}} = \sqrt[3]{\frac{5 \cdot M_{t_3}}{\tau_{dt}}} = 10,07 \text{ cm}, \text{ usvaja se } d = 11 \text{ cm}.$$

**ZADATAK BR. 20**

Odrediti  $\tau_{\max}$  grede <sup>prstenastog</sup> poprečnog preseka, odnosa  $\psi = \frac{r}{R} = \frac{1}{2}$ , ako je  $R = 100$  mm, a  $M_t = 3$  kNm.

$$\tau_{\max} = \frac{M_t}{W_0}$$

$$W_0 = \frac{I_0}{R} = \frac{\frac{\pi R^4}{2} - \frac{\pi r^4}{2}}{R} = \frac{\pi R^3}{2} (1 - \psi^4)$$

$$\tau_{\max} = \frac{2 M_t}{\pi R^3 (1 - \psi^4)} \approx 200 \text{ N/cm}^2$$

**ZADATAK BR. 21**

Dimenzionisati gredu kružno-prstenastog poprečnog preseka, odnosa  $\psi = \frac{r}{R} = \frac{1}{2}$ , ako prima moment uvijanja  $M_t = 2 \text{ kNm}$  i ukoliko je  $\tau_{dt} = 320 \text{ N/cm}^2$ .

$$\tau_{\max} = \frac{M_t}{W_0} = \frac{M_t}{\frac{\pi R^3 (1-\psi^4)}{2}} = \frac{2M_t}{\pi R^3 (1-\psi^4)} \leq \tau_{dt}$$



$$\rightarrow R \geq \sqrt[3]{\frac{2M_t}{\pi \tau_{dt} (1-\psi^4)}} = 7,4 \text{ cm}; \text{ usvaja se } R = 8 \text{ cm}$$

**ZADATAK BR. 22**

Transmisiono vratilo predaje snagu  $15 \text{ kNm/s}$  pri  $100 \text{ ob/min}$ .

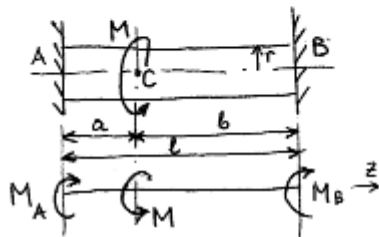
Koliki je  $\tau_{t\max}$  ako je vratilo kružno-prstenastog preseka

$$\frac{D}{d} = \frac{120 \text{ mm}}{70 \text{ mm}} ?$$

$$\tau_{\max} = \frac{M_t}{W_0} = \frac{\frac{30 \text{ P}}{\pi n}}{\frac{\pi D^3 (1-\psi^4)}{32}} = 470 \text{ N/cm}^2$$


**ZADATAK BR. 23**

Štap kružnog preseka, poluprečnika  $r$ , uklješten je između dva zida na razmaku  $l$ . Štap je opterećen obrtnim momentom  $M$ . Odrediti momente uklještenja  $M_A$  i  $M_B$  i ugao uvijanja između preseka A i C.



Odstranimo uklještenja i zamenimo njihov uticaj na štap momentima  $M_A$  i  $M_B$ . Uslov ravnoteže je:

$$\sum \hat{M}_2 = 0 : M_A - M + M_B = 0$$

Postoje dve nepoznate pa je zadatak jedanput statički neodređen.

Geometrijski uslov je da je ugao uvijanja između krajnjih preseka jednak nuli:

$$\theta_{A-B} = 0 \Rightarrow \frac{1}{GI_0} \int_0^l M_t dz = 0 \Rightarrow \int_0^l M_t dz = 0$$

$$\int_0^a M_t dz + \int_0^b M_t dz = M_A \cdot a + (M_A - M) \cdot b = 0 \Rightarrow M_A = \frac{M \cdot b}{l}, M_B = \frac{M \cdot a}{l}$$

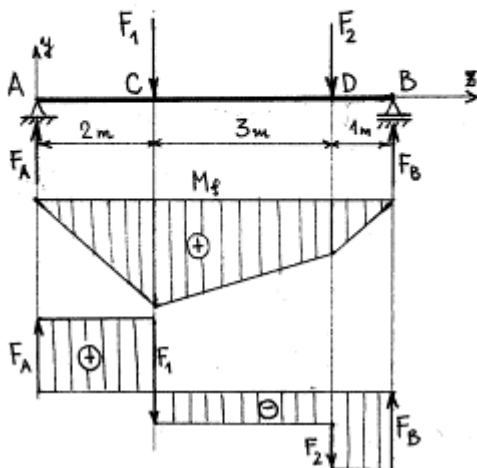
$$\theta_{A-C} = \frac{1}{GI_0} \int_0^a M_t dz = \frac{M_A \cdot a}{GI_0} = \frac{M \cdot a \cdot b}{GI_0 \cdot l}$$

## 3. SAVIJANJE (FLEKSIJA)

$$\begin{aligned} \sigma_x &= \frac{M_f}{I_x} \cdot y && - \text{I glavna g-na savijanja} \\ k &= \frac{1}{R_k} = \frac{M_f}{EI_x} && - \text{II glavna g-na savijanja} \\ \varphi &= \frac{TS_x^{\text{obs}}}{I_x \cdot b(y)} && - \text{III glavna g-na savijanja} \end{aligned}$$

## ZADATAK BR. 24

Prosta greda AB, raspona  $l = 6\text{ m}$ , opterećena je teretima  $F_1 = 40\text{ kN}$  i  $F_2 = 20\text{ kN}$ . Dimenzionisati čeličnu gredu kružnog poprečnog preseka ako je dozvoljeni napon na savijanje  $\sigma_{df} = 120\text{ MPa}$ .

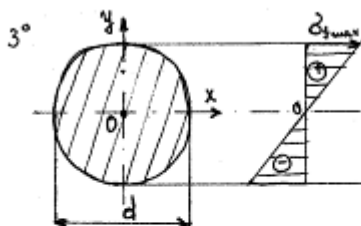


$$\begin{aligned} 1^\circ \sum Y = 0; & F_A - F_1 - F_2 + F_B = 0 \\ & F_A + F_B = F_1 + F_2 = 60 \end{aligned}$$

$$\begin{aligned} \sum \vec{M}_A = 0; & F_1 \cdot 2 + F_2 \cdot 5 - F_B \cdot 6 = 0 \\ & F_B = \frac{2F_1 + 5F_2}{6} = \frac{2 \cdot 40 + 5 \cdot 20}{6} \end{aligned}$$

$$\begin{aligned} F_B &= 30\text{ kN} \\ F_A &= 30\text{ kN} \end{aligned}$$

$$\begin{aligned} 2^\circ M_A &= M_B = 0 \\ M_C &= F_A \cdot 2 = 60\text{ kNm} \\ M_D &= F_B \cdot 1 = 30\text{ kNm} \\ M_{f_{\max}} &= 60\text{ kNm} = 60 \cdot 10^5\text{ Ncm} \end{aligned}$$



$$\sigma_{f_{\max}} = \frac{M_{f_{\max}}}{I_x} \cdot y_{\max} \leq \sigma_{df};$$

$$W_x = \frac{I_x}{y_{\max}} \Rightarrow \sigma_{f_{\max}} = \frac{M_{f_{\max}}}{W_x} \leq \sigma_{df}; \Rightarrow W_x \geq \frac{M_{f_{\max}}}{\sigma_{df}} = \frac{60 \cdot 10^5}{120 \cdot 10^6 \cdot 10^{-4}} = 500\text{ cm}^3$$

$$y_{\max} = \frac{d}{2}; I_x = \frac{\pi d^4}{64}; W_x = \frac{\pi d^3}{32};$$

$$d \geq \sqrt[3]{\frac{32 W_x}{\pi}} \Rightarrow d \geq 17,2\text{ cm}; \text{ USVAJA SE } d = 180\text{ [mm]}$$

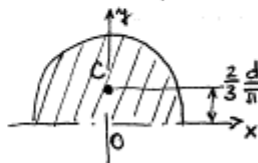
4° Provera:

$$\tau_{df} = 0,8 \delta_{df} \Rightarrow \sigma_{df} = 96 \text{ MPa}; \tau_{max} \leq \tau_{df}$$

$$\delta_{f_{max}} = \frac{M_{f_{max}}}{W_x} = \frac{M_f}{\frac{\pi d^3}{32}} = \frac{32 \cdot M_f}{\pi d^3} = \frac{32 \cdot 60 \cdot 10^5}{\pi \cdot 18^3} = 10479,33 \frac{\text{N}}{\text{cm}^2} = 10,48 \cdot 10^7 [\text{Pa}]$$

$$\delta_{f_{max}} = 104,8 \text{ MPa}$$

$$\tau_{max} = \frac{T_{max} \cdot S'_x}{I_x \cdot b}; \quad I_x = \frac{\pi d^4}{64}, \quad b = d, \quad T_{max} = 30 \text{ kN} = 30 \cdot 10^3 \text{ N}$$



$$S'_x = S_x^{1/2} = \frac{d^3}{12} = \frac{18^3}{12} = 486 \text{ cm}^3$$

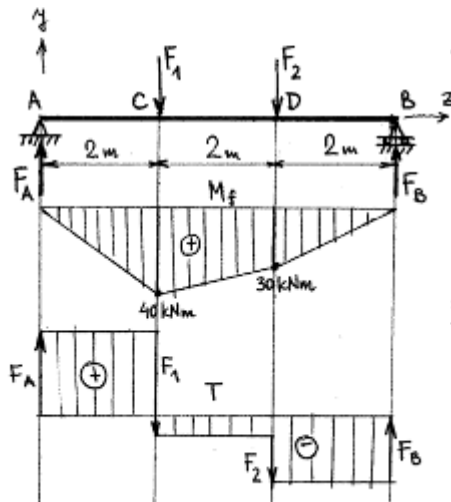
$$I_x = 5152,9973 \text{ cm}^4; \quad b = d = 18 \text{ cm}$$

$$\tau_{max} = 157,19 \frac{\text{N}}{\text{cm}^2} = 157,19 \cdot 10^4 \frac{\text{N}}{\text{m}^2} = 1,5719 \text{ MPa} \leq \tau_{df}$$

**ZADATAK BR. 25**

Prosta greda AB, raspona  $l = 6\text{ m}$ , opteređena je silama  $F_1 = 25\text{ kN}$  i  $F_2 = 10\text{ kN}$ . Dimenzionisati čeličnu gredu pravougaonog poprečnog preseka, odnosa strana  $b : h = 1 : 2$ , ako je  $\sigma_{df} = 120\text{ MPa}$ . Da li bi bio bolje iskorišćen „I” profil umesto pravougaonog popr. preseka?

$$F_1 = 25\text{ kN}, F_2 = 10\text{ kN}, \frac{b}{h} = \frac{1}{2}, \sigma_{df} = 120\text{ MPa}$$



$$1^\circ \sum Y = 0 :$$

$$F_A - F_1 - F_2 + F_B = 0 \Rightarrow F_A + F_B = 35$$

$$\sum \vec{M} = 0 :$$

$$F_1 \cdot 2 + F_2 \cdot 4 - F_B \cdot 6 = 0 \Rightarrow F_B = \frac{2F_1 + 4F_2}{6}$$

$$F_B = 15\text{ kN}; F_A = 20\text{ kN}$$

$$2^\circ M_A = M_B = 0$$

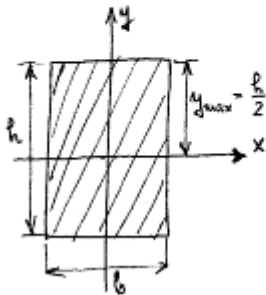
$$M_C = F_A \cdot 2 = 40\text{ kNm}$$

$$M_D = F_B \cdot 2 = 30\text{ kNm}$$

$$M_{f_{\max}} = 40\text{ kNm} = 40 \cdot 10^5\text{ Nm}$$

$$\sigma_{df} = 120\text{ MPa} = 120 \cdot 10^5 \frac{\text{N}}{\text{m}^2} = 120 \cdot 10^2 \frac{\text{N}}{\text{cm}^2}$$

$$\sigma_{f_{\max}} = \frac{M_{f_{\max}}}{W_x} \leq \sigma_{df} \Rightarrow W_x \geq \frac{M_{f_{\max}}}{\sigma_{df}} \Rightarrow W_x \geq \frac{1000}{3} [\text{cm}^3]$$



$$W_x = \frac{I_x}{y_{\max}} = \frac{\frac{bh^3}{12}}{\frac{h}{2}} = \frac{bh^2}{6}; \frac{b}{h} = \frac{1}{2} \Rightarrow h = 2b$$

$$W_x = \frac{b(2b)^2}{6} = \frac{2}{3}b^3$$

$$\frac{2}{3}b^3 \geq \frac{1000}{3} \Rightarrow b^3 \geq 500 \Rightarrow b \geq 7,937\text{ cm}$$

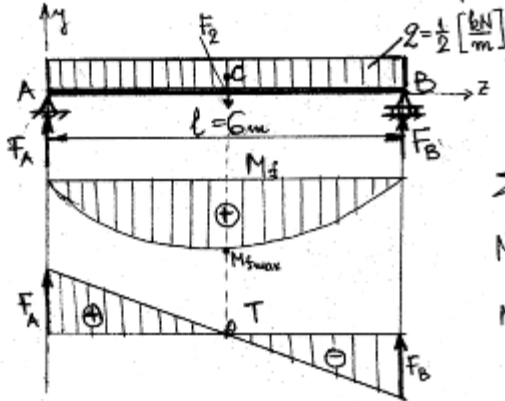
$$\text{USVAJA SE: } b = 8\text{ [cm]} \Rightarrow h = 16\text{ [cm]}$$

Za profilisani I nosač, iz tablice (str. 201), na osnovu vrednosti za  $W_x = 354\text{ cm}^3$  (prva veća vrednost od prethodno dobijene  $W_x = 341,33\text{ cm}^3$ ), usvaja se profil I24 ( $A = 46,1\text{ cm}^2$ ). Radi utvrđivanja uštede potrebno je uporediti površine poprečnih preseka:

$$\frac{A_{\square} - A_{\text{I}}}{A_{\square}} \cdot 100\% = \frac{128 - 46,1}{128} \cdot 100 = 64\% \text{ uštede u materijalu}$$

**ZADATAK BR. 26**

Dimenzionisati čeličnu gredu kvadratnog poprečnog preseka, dužine  $l = 6\text{ m}$ , ako je opterećena jednolikim opterećenjem  $q = \frac{1}{2} [\text{kN/m}]$ . Dozvoljeni napon iznosi  $\sigma_{df} = 1000 \text{ N/cm}^2$ .

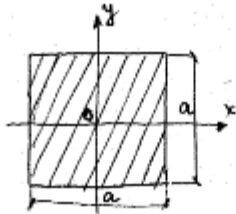


$$F_2 = q \cdot l = \frac{1}{2} \cdot 6 = 3 [\text{kN}]$$

$$\sum Y = 0 \Rightarrow F_A = F_B = \frac{F_2}{2} = 1,5 [\text{kN}]$$

$$M_{jmax} = M_c = F_A \cdot \frac{l}{2} - q \cdot \frac{l}{2} \cdot \frac{l}{4} = 2,25 \text{ kNm}$$

$$M_{smax} = 2,25 \cdot 10^5 \text{ Ncm}$$



$$\sigma_{smax} = \frac{M_{smax}}{W_x'} \leq \sigma_{df}$$

$$W_x = \frac{I_x}{y_{max}} = \frac{\frac{a^4}{12}}{\frac{a}{2}} = \frac{a^3}{6}$$

$$\frac{a^3}{6} \geq \frac{M_{smax}}{\sigma_{df}} \Rightarrow a \geq \sqrt[3]{6 \frac{2,25 \cdot 10^5}{10^3}} \approx 11,52 \text{ cm}$$

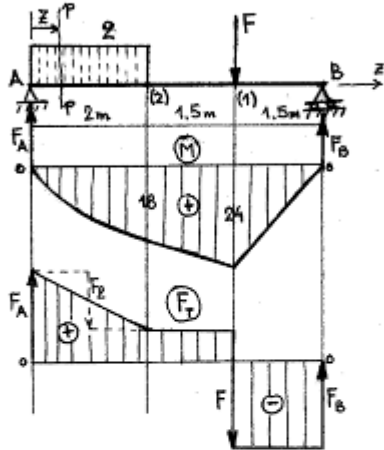
$$\text{USVAJA SE: } \underline{a = 12 \text{ cm}}$$

**ZADATAK BR. 27**

Prosta greda opterećena je koncentrisanom silom  $F = 20 \text{ kN}$  i jednako raspodeljenim kontinualnim opterećenjem  $q = 5 \text{ kN/m}$ .

Dimenzionisati prostu gredu ako je  $\sigma_{df} = 12 \text{ kN/cm}^2$  i ako je ona :

- kružnog poprečnog preseka ;
- pravougaonog poprečnog preseka odnosa stranica  $b/h = 2/3$  ;
- standardni „I“ profil .



$$\sum Y = 0: F_A + F_B - 2q - F = 0$$

$$F_A + F_B = 30$$

$$\sum M_B = 0: 5F_A - 2q \cdot 4 - 1.5F = 0$$

$$5F_A = 40 + 30 \Rightarrow F_A = 14 \text{ kN}$$

$$F_B = 16 \text{ kN}$$

$$M_1 = F_B \cdot 1.5 = 16 \cdot 1.5 = 24 \text{ kNm} = M_{f_{\max}}$$

$$M_2 = F_B \cdot 3 - F \cdot 1.5 = 48 - 30 = 18 \text{ kNm}$$

$$M_s(p-p) = F_A \cdot z - q \cdot z \cdot \frac{z}{2} = F_A \cdot z - \frac{1}{2} q z^2 \quad |_{0 \leq z \leq 2\text{m}}$$

$$\frac{dM_s}{dz} = F_A - qz = 0, \quad z = \frac{F_A}{q} = \frac{14}{5} = 2.8 \text{ m}$$

$$\text{a) } W_x = \frac{\pi d^3}{32}, \quad \sigma_{f_{\max}} = \frac{M_{f_{\max}}}{W_x} \leq \sigma_{df} \Rightarrow W_x \geq \frac{M_{f_{\max}}}{\sigma_{df}}, \quad \frac{\pi d^3}{32} \geq \frac{24 \cdot 100}{12}$$

$$d \geq \sqrt[3]{\frac{200 \cdot 32}{\pi}}, \quad d \geq 12.68 \text{ cm}; \quad \text{usvaja se } d = 13 \text{ cm}$$

$$\text{b) } W_x = \frac{bh^2}{6}, \quad W_x \geq \frac{M_{f_{\max}}}{\sigma_{df}} \therefore \frac{bh^2}{6} \geq \frac{24 \cdot 100}{12} \Rightarrow \frac{2}{3} h \cdot \frac{h^2}{6} \geq \frac{24 \cdot 100}{12}$$

$$h^3 \geq 1800, \quad h \geq \sqrt[3]{1800} \Rightarrow h \geq 12.15 \text{ cm}$$

$$b = \frac{2}{3} h = \frac{2}{3} \cdot 12.15 = 8.1 \text{ cm}$$

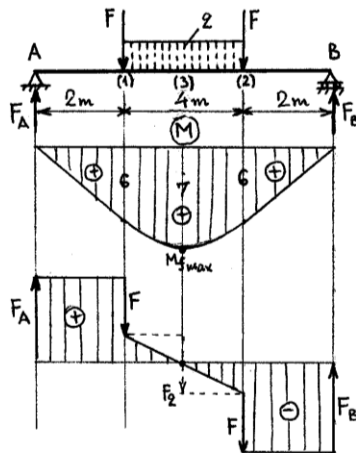
$$\text{c) } W_x \geq \frac{M_{f_{\max}}}{\sigma_{df}}; \quad W_x \geq \frac{24 \cdot 100}{12} \Rightarrow W_x \geq 200 \text{ cm}^3$$

$$\text{usvaja se } I20: \quad W_x = 214 \text{ cm}^3, \quad I_x = 2140 \text{ cm}^4$$

**ZADATAK BR. 28**

Greda  $\overline{AB}$ , raspona 8 m, poprečnog preseka predstavljenog na slici, opterećena je teretima  $F = 2 \text{ kN}$  i  $q = 0,5 \text{ kN/m}$ .

- Odrediti najveće normalne napone (ivične) u opasnom preseku grede i nacrtati dijagram normalnog napona.
- U preseku grede na udaljenju 1,5 m od levog oslonca odrediti normalni i tangencijalni napon u tačkama preseka na rastojanju 10 cm od neutralne ose.
- U tim tačkama (zadatak pod b) odrediti veličinu glavnih napona.



$$\sum Y = 0: F_A + F_B = 2F + 4q$$

$$\sum M_B = 0: F_A \cdot 8 - F \cdot 6 - 4q \cdot 4 - F \cdot 2 = 0$$

Simetrija je potpuna, pa je:

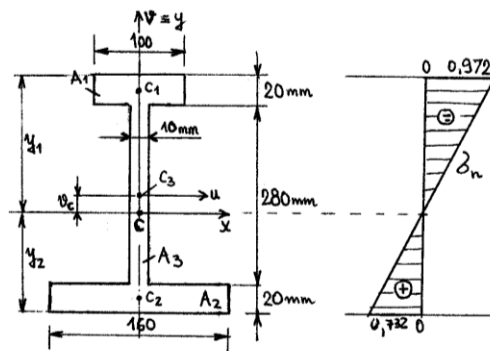
$$F_A = F_B = F + 2q = 2 + 2 \cdot 0,5 \Rightarrow F_A = F_B = 3 \text{ kN}$$

$$M_1 = M_2 = F_A \cdot 2 = 3 \cdot 2 \Rightarrow M_1 = M_2 = 6 \text{ kNm}$$

$$M_3 = F_A \cdot 4 - 2F - 2q \cdot 1 = 3 \cdot 4 - 2 \cdot 2 - 2 \cdot 0,5 \cdot 1 = 12 - 4 - 1 = 7 \text{ kNm}$$

$$M_3 = 7 \text{ kNm} = M_{fmax}$$

(opasan presek)



$$a) A = A_1 + A_2 + A_3 = 10 \cdot 2 + 16 \cdot 2 + 28 \cdot 1$$

$$A = 80 \text{ cm}^2$$

$$y_c = \frac{\sum A_i y_i}{\sum A_i} = \frac{20 \cdot 15 - 32 \cdot 15}{80} = -\frac{9}{4} \text{ cm}$$

$$y_c = -2,25 \text{ cm}$$

$$y_1 = 16 + y_c = 16 + 2,25 = 18,25 \text{ cm}$$

$$y_2 = 16 - y_c = 16 - 2,25 = 13,75 \text{ cm}$$

$$I_x = \left( \frac{2^3 \cdot 10}{12} + 2 \cdot 10 \cdot 17,25^2 \right) + \left( \frac{2^3 \cdot 16}{12} + 2 \cdot 16 \cdot 12,75^2 \right) + \left( \frac{1 \cdot 28^3}{12} + 1 \cdot 28 \cdot 2,25^2 \right)$$

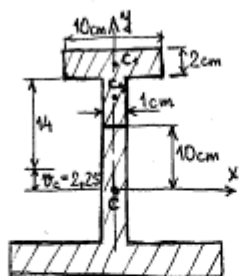
$$I_x = 13141,667 \text{ cm}^4$$

$$W_{x1} = \frac{I_x}{y_1} = \frac{13141,667}{18,25} = 720,1 \text{ cm}^3; \quad W_{x2} = \frac{I_x}{y_2} = \frac{13141,667}{13,75} = 955,76 \text{ cm}^3$$

$$\sigma_{max} = \frac{M_{fmax}}{W_{xmin}} = \frac{M_{fmax}}{W_{x1}} = \frac{7 \cdot 100}{720,1} = 0,972 \frac{\text{kN}}{\text{cm}^2}$$

$$\sigma_{min} = \frac{M_{fmax}}{W_{xmax}} = \frac{M_{fmax}}{W_{x2}} = \frac{7 \cdot 100}{955,76} = 0,732 \frac{\text{kN}}{\text{cm}^2}$$

b)  $M(z=1,5) = F_A \cdot 1,5 = 3 \cdot 1,5 = 4,5 \text{ kNm}$  ,  $F_T = 3 \text{ kN}$



$$\delta = \frac{M}{I_x} \cdot y = -\frac{4,5 \cdot 100}{13141,667} \cdot 10 = -0,343 \frac{\text{kN}}{\text{cm}^2}$$

$$q = \frac{F_T \cdot S_x'}{I_x \cdot \xi} ; \quad \xi = 1 \text{ cm}$$

$$S_x' = 20 \cdot 17,25 + 1 \cdot (2,25 + 14 - 10) \cdot \left[10 + \frac{1}{2}(2,25 + 14 - 10)\right]$$

$$S_x' = 427 \text{ cm}^3$$

$$q = \frac{3 \cdot 427}{13141,667 \cdot 1} = 0,0975 \frac{\text{kN}}{\text{cm}^2}$$

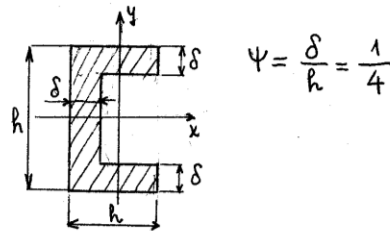
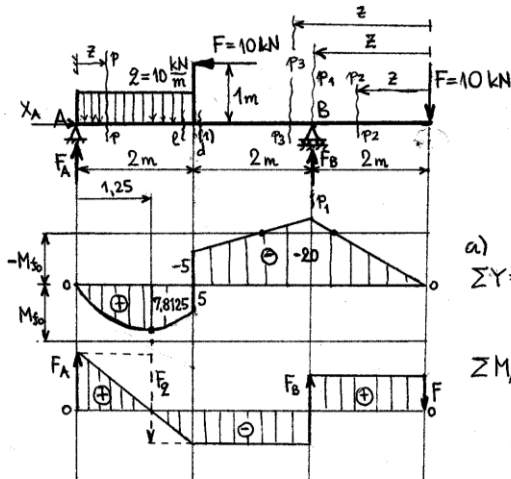
c)  $\delta_{1/2} = \frac{\delta}{2} \pm \frac{1}{2} \sqrt{\delta^2 + 4q^2} = \frac{1}{2} \left( -0,343 \pm \sqrt{0,343^2 + 4 \cdot 0,0975^2} \right)$

$$\delta_{1/2} = \frac{1}{2} \left( -0,343 \pm 0,395 \right) \Rightarrow \delta_1 = \frac{1}{2} \left( -0,343 - 0,395 \right) = -0,369 \frac{\text{kN}}{\text{cm}^2}$$

$$\delta_2 = \frac{1}{2} \left( -0,343 + 0,395 \right) = 0,026 \frac{\text{kN}}{\text{cm}^2}$$

**ZADATAK BR. 29**

- Za nosač na slici odrediti otpore oslonaca i nacrtati statičke dijagrame.
- Dimenzionisati nosač prema najvećem momentu savijanja ako je dozvoljeni napon na savijanje  $\sigma_{df} = 10 \text{ kN/cm}^2$ .
- Izračunati normalni i tangencijalni napon u preseku  $p_1-p_1$ , a u tačkama na udaljenju  $h/4$  od gornje ivice profila, visine  $h$ .
- Dimenzionisati nosač prema momentu savijanja koji iznosi 60% od najvećeg momenta i osnovni nosač pojačati lamelama.



$$\begin{aligned} \sum Y = 0: F_A + F_B &= F + 2q = 30 \\ F_A + F_B &= 30 \\ \sum M_A = 0: F_B \cdot 4 - 6F + 1 \cdot F - 1 \cdot 2 \cdot 2 &= 0 \\ F_B \cdot 4 &= 70 \\ F_B &= 17,5 \text{ kN}; F_A = 12,5 \text{ kN} \end{aligned}$$

$$\sum X_A = 0: X_A = F = 10 \text{ kN}$$

$$M_B = -2F = -20 \text{ kNm}; M_1^l = -4F + 2F_B + 1 \cdot F = 5 \text{ kNm}$$

$$M_1^d = -4F + 2F_B = -5 \text{ kNm}$$

$$M_{p-p} = F_A \cdot z - 2z \cdot \frac{1}{2} z = F_A \cdot z - \frac{1}{2} 2z^2$$

$$\frac{dM_{p-p}}{dz} = F_A - 2z = 0 \Rightarrow z = \frac{F_A}{2} = \frac{12,5}{2} = 1,25 \text{ m};$$

$$M(z=1,25) = 12,5 \cdot 1,25 - 10 \cdot \frac{1,25^2}{2} = 7,8125 \text{ kNm}$$

$$b) I_x = \frac{h^4}{12} - \frac{(h-\delta)(h-2\delta)^3}{12} = \frac{h^4}{12} [1 - (1-\psi)(1-2\psi)^3] = \frac{29}{12 \cdot 32} h^4$$

$$W_x = \frac{I_x}{y_{\max}} = \frac{29 \cdot h^4}{12 \cdot 32 \cdot \frac{h}{2}} = \frac{29}{6 \cdot 32} h^3$$

$$M_{f\max} = |-20| \text{ kNm}$$

$$\sigma_f = \frac{M_{f\max}}{W_x} \leq \sigma_{df} \Rightarrow W_x \geq \frac{M_{f\max}}{\sigma_{df}}, \frac{29 h^3}{6 \cdot 32} \geq \frac{20 \cdot 100}{10}$$

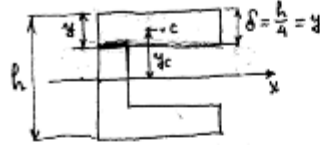
$$h \geq \sqrt[3]{\frac{6 \cdot 32 \cdot 200}{29}}, h \geq 10,98 \text{ cm}$$

$$\text{USVAJA SE } h = 11 \text{ cm}$$

$$W_x = \frac{29 \cdot h^3}{6 \cdot 32} = \frac{29 \cdot 11^3}{6 \cdot 32} = 201,036 \text{ cm}^3$$

$$c) M_{f_{P_1-P_1}} = -20 \text{ kNm}, \quad \delta/h = 1/4, \quad \delta = \frac{h}{4} = \frac{11}{4} \text{ cm}; \quad y = \frac{h}{4} = \frac{11}{4} \text{ cm}$$

$$\delta_{f_B} = \frac{M_{f_B}}{I_x} \cdot y = \frac{20 \cdot 100}{\frac{29 \cdot 11^4}{12 \cdot 32}} \cdot \frac{11}{4} = 4,97 \frac{\text{kN}}{\text{cm}^2}$$



$$F_{T_B} = 10 \text{ kN};$$

$$S'_x = A \cdot y_c = h \delta \left( \frac{h}{2} - \frac{\delta}{2} \right) = h \frac{h}{4} \left( \frac{h}{2} - \frac{h}{8} \right) = \frac{3}{32} h^3$$

$$\xi_1 = h, \quad \xi_2 = \frac{h}{4} = \delta$$

$$\sigma_{B_1} = \frac{F_{T_B} \cdot S'_x}{I_x \xi_1} = \frac{10 \cdot \frac{3 \cdot 11^3}{32}}{\frac{29 \cdot 11^4}{12 \cdot 32} \cdot 11} = \frac{10 \cdot 3 \cdot 12}{29 \cdot 11 \cdot 11} = 0,1025 \frac{\text{kN}}{\text{cm}^2}$$

$$\sigma_{B_2} = \frac{F_{T_B} \cdot S'_x}{I_x \xi_2} = 0,41 \frac{\text{kN}}{\text{cm}^2}$$

$$d) M = 0,6 M_{f_{\max}} = 0,6 \cdot 20 = 12 \text{ kNm}$$

$$W'_x = \frac{M}{\delta_{df}} = \frac{12 \cdot 100}{10} = 120 \text{ cm}^3$$

$$W'_x = 120 = \frac{29 \cdot h^3}{6 \cdot 32}, \quad h^3 = \sqrt[3]{\frac{6 \cdot 32 \cdot 120}{29}} = 9,26 \text{ cm}$$

usvaja se  $h_0 = 9,3 \text{ cm}$

$$W_{x_0} = \frac{29 h_0^3}{6 \cdot 32} = \frac{29 \cdot 9,3^3}{6 \cdot 32} = 121,49 \text{ cm}^3$$

$$M_{f_0} = W_{x_0} \delta_{df} = 121,49 \cdot 10 = 12,149 \text{ kNm}$$

Usvaja se širina lamele  $b_e = h_0 = 9,3 \text{ cm}$

Otporni moment lamela:

$$\Delta W_x^l = W_x - W_{x_0} = 201,036 - 121,49 = 79,546 = A_e \cdot h_0 = \delta_e \cdot b_e \cdot h_0$$

$$79,546 = \delta_e \cdot h_0 \cdot h_0 \rightarrow \delta_e = \frac{79,546}{9,3 \cdot 9,3} = 0,9197 \text{ cm}; \text{ usvaja se } \delta_e = 1 \text{ cm}$$

$$M_{f_{P_2-P_2}} = -F \cdot z = -M_{f_0}, \quad z = \frac{M_{f_0}}{F} = \frac{12,149}{10} = 1,2149 \text{ m}$$

$$M_{f_{P_3-P_3}} = -F \cdot z + F_B (z-2), \quad z = \frac{22,851}{7,5} = 3,0468 \text{ m}$$

$$L = 3,0468 - 1,2149$$

$$L = 1,8319 \text{ m}$$

