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Original article

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PERFORMANCE CALCULATIONS OF ELECTROSTATIC PRECIPITATORS

Abstract: *Electrostatic precipitator (ESP), as a device used to decrease the pollution content in a flowing gas using an electrostatic force, can be designed to run at any desired efficiency. An electrostatic precipitator is highly efficient in collecting the nanoparticles that cannot be removed with the help of mechanical separators or wet scrubbers. The particle charging, migration velocity of charged particles and collection efficiency are described in this review, to show that many factors influence these three core values, which are critical to the reliability and performance of electrostatic precipitators.*

Keywords: electrostatic precipitator, collection efficiency, migration velocity, particle charging

INTRODUCTION

Particles contained in gases are collected using electrostatic precipitation (ESP) after passing through a strong electric field. When particles go through the electric field, they acquire electric charges and are attracted to collection electrodes. The particles are deposited on collection plates where they lose their charge. The collected material is periodically removed by cleaning mechanisms as it accumulates [1].

COLLECTION EFFICIENCY

The schematic diagram of a dust stream flowing through an ESP is shown in Figure 1. Uniform gas velocity v throughout the cross-section is assumed. The velocity of a charged particle suspended in a gas under the influence of an electric field is known as migration (or drift, terminal, settling) velocity, w.

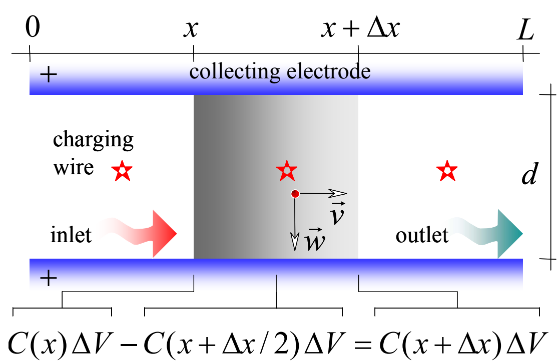

Figure 1. *Masses balance in ESP – top view*

The collection efficiency of an ESP can be derived by setting the masses balance on the input and output of two-dimensional fluid flow between two parallel plates. Let $C(x)$ denote the concentration of dust particles that is constant in time, then

$$[C(x)-C(x+\Delta x)]h\,d\,v\Delta t = C\left(x+\frac{\Delta x}{2}\right)h\,2\Delta x\,w\Delta t \quad (1)$$

The total height of ESP is h, i.e. elementary volume is $\Delta V = hd\,\Delta x$.

In a limiting case, when Δx tends to zero, (1) is reduced to the linear differential equation of the first order,

$$\frac{\mathrm{d}C(x)}{\mathrm{d}x} = -\frac{2h\,w}{h\,d\,v}C(x)\,. \quad (2)$$

Since S and Q are an area of the collection electrode and volumetric flow rate of a fluid, respectively,

$$S = 2h\,L\,,\ Q = v\,h\,d\,, \quad (3)$$

the solution of the above equation can be put in the form

$$C(x) = C_0\,e^{-\frac{wS}{Q}\frac{x}{L}},$$

where $C_0 = C(0)$ is the inlet concentration. Also, let $C_L = C(L)$ be the outlet concentration, then the collection efficiency is

$$\eta = \frac{C_0 - C_L}{C_0} = 1 - e^{-\frac{wS}{Q}}, \quad (4)$$

which is known as Deutsch-Anderson equation [2, 3].`

THEORETICAL MIGRATION VELOCITY OF CHARGED PARTICLES

The motion of a particle in fluids is an extremely complex problem. Suppose that a particle has no angular velocity. Further, suppose that forces due to collisions, wall contact, friction, diffusion can be neglected. There remain only drag force, buoyancy force and inertial force, due to particle translational motion, and gravitational and electrostatic forces due to existing fields [4]

The theoretical value of velocity is calculated by a force balance between the electrostatic force attracting the particle toward the collecting electrode and the viscous forces impeding its travel through the gas. Note that the effect of buoyancy can be ignored because the density of particles is much greater than the density of carrier gas. For the drag force, we assume that the particles are very small spheres whose radii are a . One of the fundamental results in low Reynolds hydrodynamics is the Stokes solution for steady flow past a small sphere [5].

$$F_D = 6\pi\mu a w \tag{5}$$

where μ is s the dynamic viscosity. Stokes flow, also named creeping flow, is a type of fluid flow where advective inertial forces are small compared with viscous forces. As the particle gets smaller, the medium is no longer "continuous" to the particle and each molecule is no longer invisible to the particle. Gas molecules moving around the particle may miss the particle, which is known as a "slip". When the particle size becomes comparable with the gas mean free path, slip occurs and the expression for drag must be modified accordingly [7]. The needed correction to the Stokes drag force is

$$F_D = \frac{6\pi\mu a}{C_c} w \tag{6},$$

where

$$C_c = 1 + Kn\left(\alpha + \beta e^{-\frac{\gamma}{Kn}}\right) \tag{7}$$

is dimensionless Cunningham slip correction factor [8] and $Kn = \lambda/(2a)$ is Knudsen number defined as the ratio of the molecular mean free path length to a radius of a particle. The constants α, β, γ are determined experimentally. Finally, the equation of motion of a charged spherical particle in an electric field is characterised by a differential equation, [4, 6]

$$m\frac{\mathrm{d}w}{\mathrm{d}t} + w\frac{6\pi\mu a}{C_c} = qE\ . \tag{8}$$

Taking $w(0) = 0$ as the initial condition the solution to the above equation can be readily found, i.e.

$$w = \frac{q}{m}E\tau\left(1 - \exp\left(-t/\tau\right)\right) \tag{9}$$

where τ is the relaxation time,

$$\tau = \frac{mC_c}{6\pi\mu a}\ . \tag{10}$$

Theoretically, after infinite time the particles are moved towards the collecting electrode with a velocity

$$w = \frac{qEC_c}{6\pi\mu a}\ . \tag{11}$$

Practically, the particles reach this velocity after a very short period, i.e. exponential term in (9) can be neglected. Obviously, this is the same as the inertial term ignored at the very beginning in the equation (8), i.e. as if the electric and Stokes force were immediately equalized.

SPHERICAL PARTICLE CHARGING

A dielectric sphere of a radius a and permittivity ε is placed in a region of space containing an initially uniform electric field and the permittivity of which is ε_1 , figure 2.

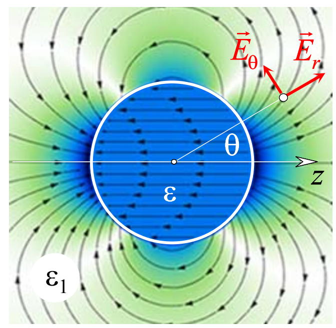

Figure 2. *Spherical particle in an initially uniform electric field*

Two charging mechanisms occur: the first is field or impact charging, and the second by ion diffusion charging. Charging the particle by ion diffusion is independent of the external electric field but only the field due to the particle charge contributes to the particle's electric field [9].

The origin of the coordinate system is taken at the center of the sphere, and the electric field is aligned along the z -axis. An electrostatic problem involving linear, isotropic, and homogeneous dielectrics reduces to finding solutions of Laplace's equation for the electric scalar potential

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\varphi}{\partial\theta}\right) = 0 \tag{12}$$

in each medium and joining the solutions in both media by means of the boundary conditions. From the azimuthal symmetry of geometry (no dependence on the azimuthal angle), the potential inside and outside can be expressed in terms of the Legendre polynomials

$$\phi = \sum(C_{1n}r^n + C_{2n}r^{-n})P_n(\cos\theta)\ . \tag{13}$$

Since the potential is finite at origin and the distant potential only has Legendre polynomial of the first order,

$$\lim_{r\to\infty}\varphi(r,\theta) = -E_0 z = -E_0 r\cos\theta \tag{14}$$

for the potential we obtain

$$\phi = \begin{cases} C_1 r\cos\theta, & r \le a \\ \left(-E_0 + \dfrac{C_2}{r^3}\right) r\cos\theta, & r \ge a \end{cases}. \quad (15)$$

The potential itself is continuous at $r = a$,

$$\varphi\big|_{r=a-} = \varphi\big|_{r=a+}. \quad (16)$$

There are no free charges on the surface and the normal component of the displacement vector is continuous across the surface. The normal component being the radial direction and we have

$$\varepsilon \frac{\partial\phi}{\partial r}\bigg|_{r=a-} = \varepsilon_1 \frac{\partial\phi}{\partial r}\bigg|_{r=a+} \quad (17)$$

Two boundary conditions yield two equations for the two unknown coefficients which are easily solved to give

$$C_1 = -\frac{3\varepsilon_1}{\varepsilon + 2\varepsilon_1} E_0, \quad C_2 = K(\varepsilon, \varepsilon_1) a^3 E_0, \quad (18)$$

where

$$K(\varepsilon, \varepsilon_1) = \frac{\varepsilon - \varepsilon_1}{\varepsilon + 2\varepsilon_1} \quad (19)$$

is Clausius-Mossotti factor, also known from Lorentz-Lorentz equation. The electric potential and field in the gas, $r \ge a$, in the vicinity of the charged particle, are

$$\phi = \underbrace{-E_0 r\cos\theta}_{\text{imposed}} + \underbrace{E_0 K(\varepsilon, \varepsilon_1) \frac{a^3}{r^2} \cos\theta}_{\text{induced}} \quad (20)$$

$$E_r = -\frac{\partial\phi}{\partial r} = \left(1 + 2K(\varepsilon, \varepsilon_1) \frac{a^3}{r^3}\right) E_0 \cos\theta \quad (21)$$

$$E_\theta = -\frac{\partial\phi}{r\,\partial r} = -\left(1 - K(\varepsilon, \varepsilon_1) \frac{a^3}{r^3}\right) E_0 \sin\theta \quad (22)$$

Since the potential from an electric dipole depends on $\cos\theta / r^2$, [10],

$$\phi_{dipole} = \frac{1}{4\pi\varepsilon_0} \frac{p}{r^3} r\cos\theta \quad (23)$$

we see that, as expected, a uniform field induces a dipole moment in the sphere.

$$\vec{p} = 4\pi\varepsilon_1 K(\varepsilon, \varepsilon_1) a^3 \vec{E}_0 \quad (24)$$

The factor (19) confirms previous knowledge: when $\varepsilon > \varepsilon_1$ the induced dipole moment and the external field are parallel [12], while they become anti-parallel when surrounding gas is not air and when $\varepsilon < \varepsilon_1$ figure 3.

For the particle in the air, $\varepsilon_1 \approx \varepsilon_0$, polarization is

$$\vec{P} = \frac{\vec{p}}{V} = \frac{3\vec{p}}{4\pi a^3} = 3\varepsilon_0 K(\varepsilon, \varepsilon_0) \vec{E}_0 \quad (25)$$

The potential inside the sphere, $r \le a$

$$\phi = -\frac{3\varepsilon_0}{\varepsilon + 2\varepsilon_0} E_0 r\cos\theta = -\frac{3\varepsilon_0}{\varepsilon + 2\varepsilon_0} E_0 z \quad (26)$$

produces a constant field in the z-direction

$$\vec{E} = -\frac{\partial\phi}{\partial z} = \frac{3\varepsilon_0}{\varepsilon + 2\varepsilon_0} \vec{E}_0 \quad (27)$$

which corresponds to a polarization

$$\vec{P} = (\varepsilon - \varepsilon_0)\vec{E} = 3\varepsilon_0 K(\varepsilon, \varepsilon_0) \vec{E}_0, \quad (28)$$

which are the results (25). The uniform external electric field induces the constant polarization inside a dielectric sphere. The induced polarization gives rise to surface charge which produces an opposing electric field.

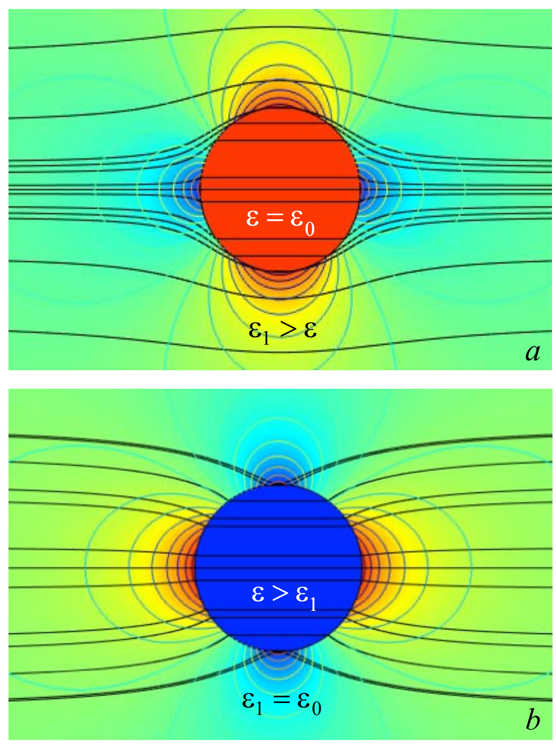

Figure 3. *Spherical dielectric particle after entering in an initially uniform electric field - field lines; a) an air bubble in a dielectric; b) a dielectric particle in an air*

The induced charge density at the surface of the sphere is

$$\sigma_b = \vec{P}\hat{n} = \vec{P}\hat{r} = 3\varepsilon_0 K(\varepsilon, \varepsilon_0) E_0 \cos\theta \quad (29)$$

The bound charges on the one hemisphere are

$$q_b = 2\pi a^2 \int_0^{\pi/2} \sigma_b \sin\theta \, d\theta = 3\pi a^2 \varepsilon_0 K(\varepsilon,\varepsilon_0) E_0, \qquad (30)$$

and there is an equal amount of negative charge on the other hemisphere.

The external electric field also ionizes the dielectric region surrounding the sphere. Mobile charge carriers will be driven by the electric field to charge the sphere surface with an additional time-dependent charge $q(t)$. A positive charge can only be deposited on the sphere where the radial component of the electric field is negative, due to negative surface charge on the sphere, and vice versa. The additional charge is distributed uniformly on the sphere surface and its effect on the potential is found by superposition, [11].

$$E_r = \left(1 + 2K(\varepsilon,\varepsilon_0)\frac{a^3}{r^3}\right) E_0 \cos\theta + \frac{q(t)}{4\pi\varepsilon_0 a^2} \qquad (31)$$

The two adjacent charging regions are connected on the sphere at a coordinate $r = a$ and $\theta = \theta_c$, where the radial electric field is zero, $E_r = 0$. The critical polar angle on the sphere is

$$\cos\theta_c = -\frac{q(t)}{q_s} \qquad (32)$$

where

$$q_s = 4\pi\varepsilon_0 a^2 \left(1 + 2K(\varepsilon,\varepsilon_0)\right) E_0 \qquad (32)$$

or

$$q_s = 4\pi a^2 \cdot \frac{\varepsilon}{\varepsilon + 2\varepsilon_0} 3\varepsilon_0 E_0 \qquad (33)$$

is a saturation charge [13] and S is the surface of the sphere. Special field line, that passes through the critical point, separates field lines which starting at infinity and terminate on the sphere, from field lines that go around the sphere.

The electric field inside a perfect conductor is always zero under the static situation, so the dielectric constant as a ratio of polarization density and electric field for a conductor is infinite, figure 4.

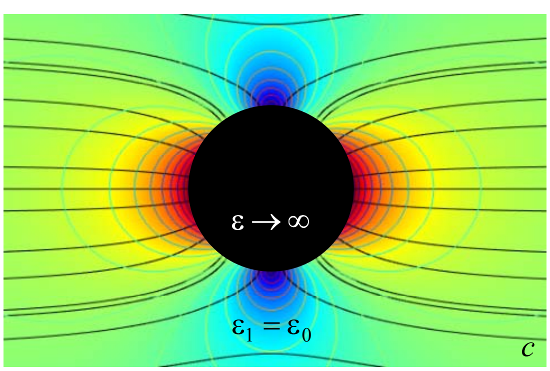

Figure 4. *A perfectly conducting particle in an air*

From the above analysis, it is quite evident that,

$$\lim_{\varepsilon\to\infty} K(\varepsilon,\varepsilon_0) = \lim_{\varepsilon\to\infty} \frac{\varepsilon - \varepsilon_0}{\varepsilon + 2\varepsilon_0} = 1 \qquad (34)$$

$$E_\theta = 0$$
$$E_r = \left(1 + 2\frac{a^3}{r^3}\right) E_0 \cos\theta \qquad (35)$$

$$q_s = 4\pi a^2 \cdot 3\varepsilon_0 E_0 \qquad (36)$$

The last expression is known as the Pauthenier equation [6, 13].

Finally, when field charging is applicable, the migration velocity is written as

$$w = \frac{2a}{\mu} \cdot \frac{\varepsilon\varepsilon_0}{\varepsilon + 2\varepsilon_0} C_c E_0^2 \qquad (37)$$

Migration velocities are difficult to estimate on a purely theoretical basis.

CALCULATION

The equation (4) provides an idea that the efficiency of filtering is essentially influenced by the surface of the electrodes and the flow velocity of the fluid. The large surface of collecting electrodes and creeping flow of fluid causes an exponential term to tend to zero, that is, efficiency tends to the unit. However, the theoretical migration velocity of the particles is very small. What's more, the calculated velocity is a hundred times smaller than that experimentally determined [6].

In practice, velocity is estimated from pilot studies or based on previous designs because particles are randomly shaped and in various sizes, electric fields are not constant and gas flows are not uniform.

If the migration velocity is known, then Equation (4) can be rearranged to give the specific collecting area (*SCA*)

$$SCA = \frac{S}{Q} = -\frac{1}{w} \ln(1 - \eta) \qquad (38)$$

Since the theoretical migration velocity is lower than the actual one, the application of the above formula yields quite satisfactory results.

CONCLUSION

Although the Deutsch-Anderson equation is widely used in the design of ESPs, its assumptions of monodisperse particles and constant migration velocity of particles in the ESP restrict its ability to provide accurate predictions. To make the Deutsch-Anderson equation more accurate, an effective migration velocity w_e can be substituted for the migration velocity w in the equation [2]. The values for this variable are usually determined using data from pilot studies.

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BIOGRAPHY

Dejan M. Petković was born in 1952 in Niš, Serbia. He graduated from the Faculty of Electronic Engineering, University of Niš, in 1976. He received the Magister of Science and Doctor of Science degrees from the Faculty of Electrical Engineering, University of Niš, in 1983 and 1992, respectively.After graduation, he joined the Faculty of Occupational Safety in Niš, where he worked as a professor of Electromagnetic radiation, until his retirement in October 2018. His current research interests include the theoretical and numerical modeling of electromagnetic problems. He has published two monographs and six textbooks. His list of publications contains more than one 150 scientific papers published in reviewed journals and international conferences proceedings.

PRORAČUN PERFORMASNI ELEKROSTATIČKIH FILTARA

Milica D. Radić, Dejan M. Petković, Dejan D. Krstić, Dejan N. Jovanović

Rezime: *Elektrostaitčki filtri (ESP) su uređaji koji se koriste za smanjenje zagađenja ukljanjajem čestica iz vazduha ili gasa korišćenjem elektrostatičke sile koji može biti dizajniran tako da radi sa željenom efikasnošću. Elektrostatički filter je visoko efikasan u sakupljanju nanočestica koje se ne mogu ukloniti uz pomoć mehaničkih separatora ili vlažnih skrubera. Naelektrisavanje čestica, brzina kretanja naelektrisanih čestica i efikasnost sakupljanja opisani su u ovom radu, kako bi pokazali da mnogi faktori utiču na ove tri osnovne vrednosti, koje su ključne za pouzdanost i performanse elektrostatičkih taložnika.*

Ključne reči: elektrostatički filtar, efikasnost sakupljanja, brzina kretanja čestica, naelektrisavanje čestica.