

BOJANA ZLATKOVIĆ¹
BILJANA SAMARDŽIĆ²¹University of Niš,
Faculty of Occupational Safety,
²University of Niš,
The Faculty of Science and
Mathematics¹ bojana.zlatkovic@znrfa.ni.ac.rs² biljana@pmf.ni.ac.rs

PETRI NETS MODELLING OF SYSTEMS WITH REDUNDANCY

Abstract: *Petri nets are very appropriate for modelling and analysis of different systems. In this paper, examples of repairable and irreparable systems modelled using Petri nets have been given. Systems with cold and hot redundancy have been analyzed, also. The redundancy is illustrated using the example of the electric power system.*

Key words: redundancy, Petri nets, reliability.

INTRODUCTION

Petri Nets are a graphical and mathematical modeling tool applicable to many systems [1-6]. They are used for describing and studying information processing systems that are characterized as being concurrent, asynchronous, distributed, parallel, nondeterministic, and/or stochastic. As a graphical tool, Petri Nets can be used for visual - communication similar to block diagrams, networks. In these nets, markers are used to simulate the system dynamics. As a mathematical tool, it is possible to set up the state equations, algebraic equations and other mathematical models which describe the behaviour of a dynamical system.

Petri nets are very appropriate for reliability modelling and analysis of different systems. Petri nets can be applied in the field of systems safety, logistic, industry, computer science, etc. They are simple, easy to expand and analysed using simulation. Petri nets are the only class of graphs allowing complete analysis of reliability [7-11]. If some of the system state coordinates are random variables, Stochastic Petri nets can be used for the analysis of such systems.

Reliability is one of the most important criteria which must be taken into consideration during all phases of system: planning, designing and exploitation. During the exploitation, the system is under influence of different factors, exterior and interior, i.e., wear, corrosion, aging, etc. These factors can change the system's characteristics and have a strong influence on its work. Also, they can change the values of system parameters and their reliability and in limited cases can bring the system to instability or failure, [12, 13].

System reliability can be increased by using redundancy. Examples of repairable and irreparable systems modelled using Petri nets have been given.

The work of the system with cold and hot redundancy was analyzed on specific examples.

SYSTEM RELIABILITY AND REDUNDANCY

The scientific and technical progress of systems results in the appearance and development of reliability theory. Complex systems should have high reliability considering the consequences of their failures

influencing further work and people security.

Increasing the reliability of the system can be achieved in two ways: 1) increasing the reliability of system components, 2) using redundancy.

For the component to better perform its function, when choosing the system components, working conditions must be taken into account: ambient temperature, humidity, pressure, mechanical loads, allowable energy losses, the required stability of parameters, etc., as well as the causes of component failure under system working conditions. The component should be used in conditions that best suit its properties and characteristics. The use of components in normal regimes reduces the intensity of aging, i.e., prolongs the period of normal operation, and also reduces the intensity of the development of potential defects that can lead to component or system failure.

Redundancy means the introduction of an additional number of elements to fulfill the given functions in the given conditions. These additional elements are called spare elements, while the basic and spare elements form a redundant group. If the elements of the system have relatively low reliability, the spare elements are connected in parallel to them and are in the same working conditions. If the elements of the system are sufficiently reliable, the spare elements take over their function upon their failure.

If during the system work, the system is not repaired and its elements are not replaced, it is considered irreparable in which the time of failure-free work refers exclusively to the period from the start of work to the first failure. Otherwise, it is repairable. With these systems, it can be assumed that there is a time of failure-free work until the first failure, from the first to the second failure, from the second to the third, etc. Repairing means replacing defective parts after which the system is restored to a new condition. It is assumed that the recovery time is a random variable with a known probability distribution function. Depending on the work regime of the spare elements, there are three types of system stand by redundancy: cold, hot and warm redundancy. In the case of hot redundancy, the spare elements are in the same operating mode as the basic element and therefore their reliability does not depend on the moment of switching on the place of the basic element, as well as on the failure of other

elements. Such spare element works in parallel with the basic element and is included only when the basic element fails.

In case of cold redundancy, the spare elements are switched off and cannot be canceled in that period, and they start working upon arrival at the place of the basic element. The redundancy is "cold" because the spare element does not work when the basic element is correct. After the repair, the basic element is put into operation again, and the spare is switched off.

If the spare elements are less loaded than in normal operating mode, this is a warm redundancy. The reliability of these elements is higher while they are in reserve than when they are included in the place of the basic element.

The analysis of hot and cold redundancy led to the conclusion that the system reliability indicators, for the same reserve group, are higher for cold redundancy, i.e., a cold redundancy is more reliable than a hot one. On the other hand, redundancy is more reliable if there is a larger reserve group. However, an increase in the reserve group causes an increase in other system parameters: weight, volume, price, etc.

THE BASIC ELEMENTS OF PETRI NETS

Petri net is a special case of a directed graph with an initial state called initial marking, M_0 . Also, Petri net represents a bipartite, weighted graph consisting of two kinds of nodes called places and transitions. Nodes are connected by arcs. Arcs are directed either from place to transition or from transition to a place. Places are denoted by circles and transitions by bars or boxes. Arcs are labeled with their weights (positive integers), so the k – weighted arc can be interpreted as a set of k parallel arcs. A place is an input place to a transition if there exists a directed arc connecting this place to the transition, and vice versa.

Each place p is marked with nonnegative integer k . It is said that the place is marked with k tokens. Marking is denoted by m – vector M , where m is the total number of places. The p th element of vector M , denoted as $M(p)$, represents the numbers of tokens in place p .

The presence or absence of a token in a place can indicate whether a condition associated with this place is true or false, for instance. At any given time instance, the distribution of tokens on places, called Petri Net marking, defines the current state of the modeled system.

In its simplest form, a Petri Net may be represented by a transition together with its input and output places. This elementary net may be used to represent various aspects of the modelled systems.

Petri Net can be defined as $PN = (P, T, I, O, M_0)$, where

1. $P = \{p_1, p_2, \dots, p_m\}$ is a finite set of places,
2. $T = \{t_1, t_2, \dots, t_n\}$ is a finite set of transitions, $P \cup T \neq \emptyset$, and $P \cap T = \emptyset$,

3. $I : (P \times T) \mapsto N$ is an input function defining directed arcs from places to transitions, where N is a set of nonnegative integers,

4. $O : (P \times T) \mapsto N$ is an output function defining directed arcs from transitions to places, and

5. $M_0 : P \mapsto N$ is the initial marking.

The following rules are used to govern the flow of tokens.

Enabling Rule: A transition t is said to be enabled if each input place p of t contains at least the number of tokens equal to the weight of the directed arc connecting p to t .

Firing Rule:

- (a) An enabled transition t may or may not fire and
- (b) a firing of an enabled transition t removes from each input place p the number of tokens equal to the weight of the directed arc connecting p to t . It also adds in each output place p the number of tokens equal to the weight of the directed arc connecting t to p .

Classical Petri nets can be used only for the analysis of qualitative system characteristics because they do not contain the time concept. It is possible to describe the logical structure of the modelled system only.

By introducing the time into Petri nets, the Timed Petri nets are obtained. They can be used for the quantitative analysis of the system too.

In most Time Petri nets, a time delay is given to the transitions. Only in some types of Petri nets, a time delay is given to the places and/or arcs. Tokens of place p become unreachable to all input transitions for a certain time period. Petri nets like this are called *Timed Places Petri nets* (TPPNs). However, it is convenient to give delays to the transitions because they represent activities that are performed on time. When transition becomes enabled it will fire after a certain time. Petri nets like this are called *Timed Transitions Petri nets* (TTPNs). If time is a random variable, we talk about Stochastic Petri nets (SPN).

PETRI NETS MODELLING OF SYSTEMS WITH FAILURES

This section deals with the simple examples of how Petri nets can be used to model systems with failures and how those failures can be removed.

The following figure shows the Petri net of a system consisting of one component that can be repaired after a failure. Note that the presence of a marker in place means that the system is working, while the absence of a marker in place indicates that the system is in a failed state. The presence of a marker in a place indicates the current state of the system and indicates that that place is activated. Also, the presence of markers in the place indicates the occurrence of failure. The transition is activated only after the expiration of the time interval L , which represents the time of proper operation of the system. The system is in the state of failure (marker in

place) until the transition is activated, i.e., after the expiration of time D .

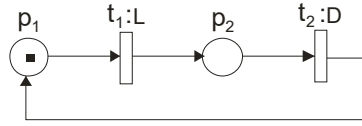


Figure 1. System with one repairable component

Figure 2 shows the Petri net of the system, which consists of two components that can be repaired independently in the event of a failure. The entire system fails if both components fail. L_i and R_i $i = 1, 2$ indicate the switching times of the appropriate transitions, i.e., the time of correct operation of the component and the time required for repair of the component if a failure occurs, respectively, while D in place p_6 indicates that the system is in a state of failure. If, during the repair of one component, another component fails, the repair of the first component is stopped due to the failure of the entire system.

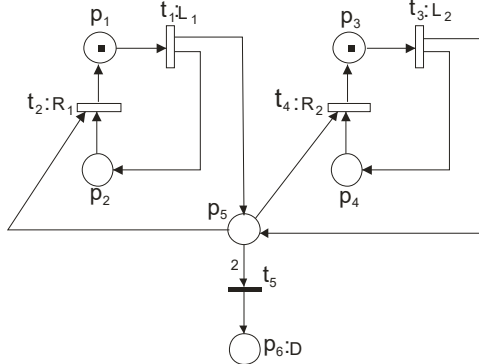


Figure 2. System with two repairable components

So far we have analysed the systems that can be repaired after a failure and these are repairable systems. However, some systems cannot be repaired, but components that have failed can be replaced with new ones. Such systems are called irreparable systems. These systems are much easier to model with Petri nets than repairable systems. Figure 3 shows a general scheme of an irreparable system with n components. In this system, if one component fails, the entire system goes into failure. The presence of a marker in place means that the system is working properly. Section 2 discussed the cold redundancy. This is an element that is included in the system in case a component fails. The spare is "cold" because the spare does not work until it replaces the failed component. Figure 4 shows a system in which the component has a working life lasting from L_1 time moments and in case of failure there are three reserves with which it can be replaced with time durations L_2, L_3, L_4 , respectively. Transition t_5 and place p_6 model system failure status and ensure that the place p_1 is unmarked. The

inhibitory arc prevents more than one marker from being sent from place p_1 to place p_2 .

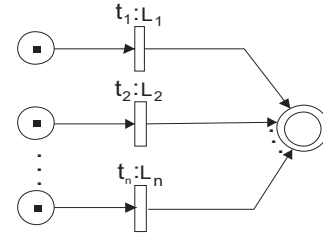


Figure 3. General scheme of an irreparable system with n components

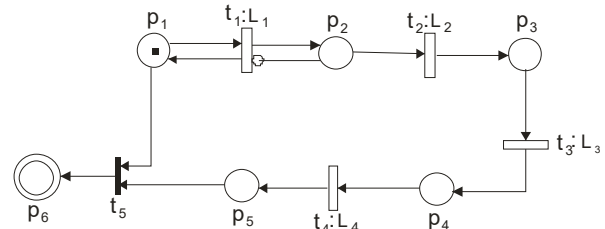


Figure 4. Scheme of the system with 3 cold redundancies

The following figure shows a scheme of a system with a hot redundancy.

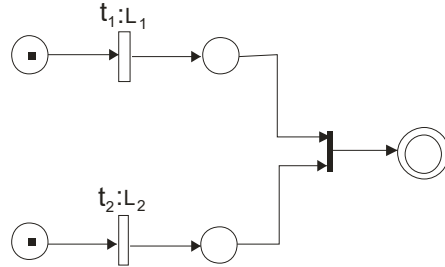


Figure 5. Scheme of the system with hot redundancy

The hot redundancy works in parallel with the component, but only turns on when the component fails. Transitions may be immediate (when enabling conditions are fulfilled) or timed with a delay > 0 . The delays may be deterministic or stochastic. This means that stochastic Petri nets can be used in this case.

In most cases, a hot redundancy is cheaper than a cold redundancy.

Cold redundancy is best suited to non-critical processes where downtime is not a big concern and human intervention is possible. However, warm redundancy design is suited to processes where time and response are important but a momentary outage is still acceptable.

The redundancy is illustrated using the example of an electric power system, [14]. From power plant E through the power lines L_1 and L_2 consumer C is powered. Consumer C is connected to the buses S . Buses S are supplied from the local source G which is connected over switch B .

The following symbols have been introduced:

- IC - supply interruption of consumer C ,
- x - malfunction at the same time of generator G and system for buses supply,
- z - malfunction of generator G and failure of switch B ,
- y - malfunction at the same time of both wiring or malfunction of power plant E ,
- v - malfunction at the same time of both wiring,
- \bar{S} - malfunction of buses S ,
- \bar{E} - malfunction of the power plant,
- \bar{G} - malfunction of the generator,
- \bar{B} - failure of a switch,
- \bar{L}_1, \bar{L}_2 - malfunction of both wirings L_1 and L_2 .

The appropriate Petri net is given in the next figure. This is an example of hot redundancy.

The presence of tokens in a place denotes that a certain event happened. By using classical Petri net, it can be determined if the failure of consumer C appeared or not. Stochastic Petri nets can be used if we want to determine the probability of failure.

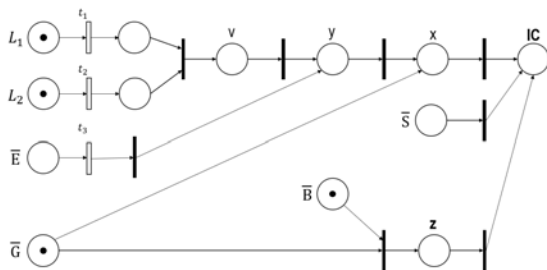


Figure 6. Petri net of electric power system

CONCLUSION

The appearance and development of reliability theory is a consequence of the scientific and technical progress of the systems. For the reliability analysis, Petri nets can be used successfully. Petri nets are the only class of graphs allowing complete analysis of reliability. In addition, Petri nets are a graphical and mathematical modeling tool applicable to many systems.

In this paper, examples of repairable and irreparable systems modelled using Petri nets have been given. The systems with cold and hot redundancy have also been analyzed.

REFERENCES

- [1] T. Murata, "Properties, analysis and applications", *Proceedings of IEEE*, 77(4):541-580, April 1989.
- [2] F. Bause, P. S. Kritzinger, "Stochastic Petri nets: An introduction to the theory", 2nd Edition, Vieweg, 2002.
- [3] R. German, C. Kelling, A. Zimmermann, G. Hommel, "TimeNET – A Toolkit for evaluating non – Markovian stochastic Petri nets".
- [4] G. Horton, "Stochastic Petri nets", *Introduction to Simulation* WS02/03 – L08.
- [5] P. J. Haas, "Stochastic Petri nets. Modeling, stability, simulation", Springer, 2002.
- [6] J. A. Borrie, "Stochastic Systems for Engineers", New York, Prentice Hall, 1996.
- [7] J. M. Nahman, "Metode analize pouzdanosti elektroenergetskih sistema", Naučna knjiga, Beograd, 1992.
- [8] W. G. Schneeweiss, "The fault tree method" (From the field of reliability and safety technology), LiLoLe – Verlag GmbH, Hagen, 1999.
- [9] W. G. Schneeweiss, "Petri nets for reliability modeling" (In the fields of engineering safety and dependability), LiLoLe – Verlag GmbH, Hagen, 1999.
- [10] Bojana M. Vidojković, "Primena Petri mreža u analizi pouzdanosti upravljačkih sistema", Magistarski rad, Elektronski fakultet, Niš, 2003.
- [11] B. Danković, B. Vidojković, "A Petri Net-Based Approach to Logic Control and a Case Study", *Proceedings of the VI International SAUM Conference on Systems, Automatic Control and Measurements*, Niš, September 1998, pp. 253-257.
- [12] M. Tomić, Adamović, "Pouzdanost u funkciji održavanja tehničkih sistema", Tehnička knjiga, Beograd, 1986.
- [13] S. Jovičić, "Osnovi pouzdanosti mašinskih konstrukcija", Naučna knjiga, Beograd, 1990.
- [14] B. M. Zlatković, B. Samardžić: "A New Approach for the System Time without Failures Determining Using Petri Nets", *Safety Engineering, Scientific Journal*, Vol. 5, No. 2 (2015), pp. 85–90, Faculty of Occupational Safety, University of Nis, doi: 10.7562/SE2015.5.02.03.

BIOGRAPHY of the first author

Bojana M. Zlatković was born in Jagodina, Serbia, in 1976. She received the B.Sc., M.Sc. and Ph.D. degrees from the Faculty of Electronic Engineering, University of Nis. She is currently working as an Associate Professor at the Faculty of Occupational Safety in Nis.



Her main areas of research include reliability, probability of stability, chaos theory, etc.

MODELIRANJE SISTEMA SA REZERVOM KORIŠĆENJEM PETRI MREŽA

Bojana Zlatković, Biljana Samardžić

Rezime: Petri mreže su veoma pogodne za modeliranje i analizu različitih sistema. U ovom radu su dati primeri popravljivih i nepopravljivih sistema modelovanih pomoću Petri mreža. Analizirani su i sistemi sa hladnom i toplom rezervom. Sistem sa rezervom je ilustrovan na primeru elektroenergetskog sistema.

Ključne reči: rezervisanje, Petri mreže, pouzdanost.