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DEJAN JOVANOVIĆ[1]
NENAD CVETKOVIĆ[2]
MIODRAG STOJANOVIĆ[3]
MARINKO BARUKČIĆ[4]
ŽELJKO HEDERIĆ[5]
VLADIMIR STANKOVIC[6]

[1,23]Faculty of Electronic Engineering, University of Niš
[4,5]J. J. Strossmayer University of Osijek, FERIT, Croatia
[6]Faculty of Occupational Safety, University of Niš

[1]dejan.jovanovic@elfak.ni.ac.rs

AN APPROXIMATE ANALYTICAL EXPRESION FOR RESISTANCE OF THE VERTICAL GROUNDING ELECTRODE PLACED IN MULTILAYERED MEDIA

Abstract: *An expression for the resistance of the grounding electrode placed in the inhomogeneous ground approximated by a finite number of homogeneous layers of constant specific conductivity has been evaluated and proposed in the paper. The expression is obtained by optimization procedure, based on processing of data sets obtained as a result of the analysis which includes using of the Green's function for the point source in multilayered soil and the Method of Moments. The approach has been applied to the characterization of a vertical electrode placed in the three-layered soil.*

Key words: Green's functions methods, Grounding, Optimization methods.

INTRODUCTION

The problem of modeling and analyzing nonhomogeneous ground is of great importance in grounding system theory, since ground structure influences the grounding systems' characteristics. The large number of publications dealing with procedures for analyzing grounding systems in non-homogeneous ground approximated by homogeneous domains has been published [1]-[3].

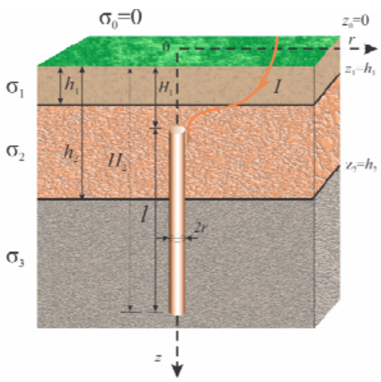

Figure 1. *Ground electrode in multilayered ground*

In this paper, a semi-analytical procedure for characterization of a vertical grounding electrode placed in the nonhomogeneous ground approximated by a finite number of homogeneous layers has been presented. The procedure includes using of the Green's functions for a point source in multilayered media obtained by solving the Poisson's i.e. the Laplace's equation for the electric scalar potential [4] and the Method of Moments [5]. Above-mentioned procedure has been applied for generating input data sets for optimization procedure based on differential evaluation (DE) [6]-[7]. The result of optimization process is analytical expression for resistance of vertical electrode placed in multilayered ground.

The complete previously described approach is applied to a vertical ground electrode placed in the three-layered ground, Fig. 1. The dimension of the electrode and electrical parameters of soil structure have been chosen based on the parameters overtaken from previously published researches and official publications. [8]-[9].

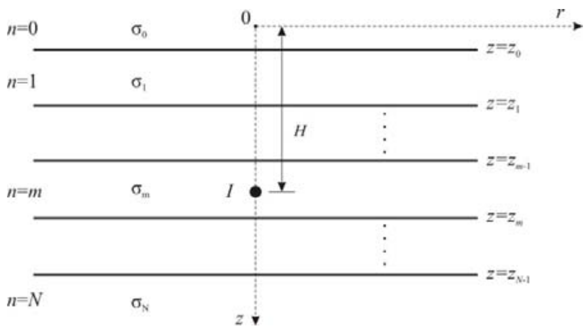

Figure 2. *Point current source in multilayered media*

PROCEDURE FOR ANALYZING MULTILAYERED GROUND

In this chapter, firstly the Green's function for point source will be presented. Afterwards, the Method of Moments will be introduced.

Green's function

The point source of current I placed in the non-homogenous domain approximated by total of $N+1$ homogenous horizontal layers of specific conductivity $\sigma_n, n=0,1,...N$ is observed, Fig. 2. The potential of the system from Fig. 2 satisfies the Laplace's, i.e. the Poisson's equation

$$\Delta\varphi=\begin{cases}0,\ n\neq m,\ n=0,1,...N\\ -\dfrac{I}{2\pi\sigma_n r}\delta(r)\delta(z-H),n=m,\ n=0,1,...N\end{cases} \quad (1)$$

In (1), r and z denote cylindrical coordinates, while δ is a one-dimensional Dirac delta function

General solution of differential equation (1) can be assumed in the form [4],

$$\varphi_{g\,nm}(r,z)=\int_0^\infty f_{nm}(z,k)J_0(kr)k\,\mathrm{d}\,k, n=0,1,...N,$$

(2a)

$$f_{nm}(z,k)=\begin{cases}A_n e^{kz}+B_n e^{-kz}, n=0,N,\ n\neq m\\ A_m e^{kz}+B_m e^{-kz},\ n=m,\ z_{m-1}\leq z\leq H\\ \left(A_m-\dfrac{I}{4\pi\sigma_m k}e^{-kH}\right)e^{kz}+\\ +\left(B_m+\dfrac{I}{4\pi\sigma_m k}e^{kH}\right)e^{-kz}, n=m,\ H\leq z\leq z_m.\end{cases} \quad (2$$

b)

In (2), φ_{nm}, $n,m=0,1,...,N$ is the potential at the points in *n*-th layer, when the source is in the layer *m*, A_n B_n, $n=0,1,...,N$ are unknown coefficients, while J_0 denotes the Bessel function of the first kind of zero order.

Unknown coefficients $A_n, B_n, n=0,1,...,N$ from (2) can be determined from the boundary conditions

$$\varphi\left(z=z_n^-\right)=\varphi\left(z=z_n^+\right), n=0,N-1, \quad (3a)$$

$$\sigma_n\frac{\partial\varphi}{\partial z}\left(z=z_n^-\right)=\sigma_{n+1}\frac{\partial\varphi}{\partial z}\left(z=z_n^+\right), n=0,N-1, \text{ and} \quad (3b)$$

$$\lim_{z\to\pm\infty}\varphi=0. \quad (3c)$$

for the potential (3a), normal component of the conducting current (3b), and the finite value of the potential (3c).

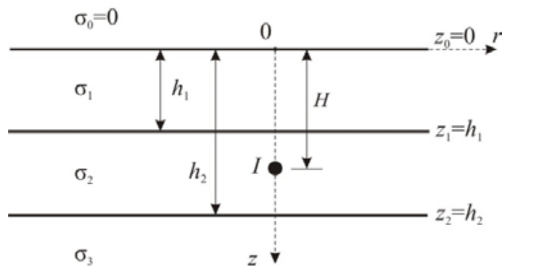

Figure 3. *Point current source in three-layered ground (layer 2)*

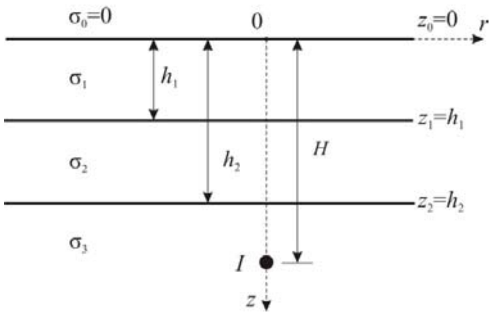

Figure 4. *Point current source in three-layered ground (layer 3)*

Three-layered soil structure

To determine parameters of a vertical ground electrode from Fig. 1, it is necessary to obtain the Green's function of a point source placed in non-homogeneous ground approximated with three horizontal homogeneous layers $n=1,2,3$ ($N=3$), while layer labeled by $n=0$ corresponds to air, ($\sigma_0=0$), as it is shown in Figs. 3-4. The Green's function can be obtained based on the general solution (3) and boundary conditions (3).

The potential in the *n*-th layer from the point source placed in layer with specific conductivity σ_2 (*m*=2), Fig. 3, is

$$\varphi_{g\,n2}(r,z)=\int_0^\infty f_{n2}(z,k)J_0(kr)k\,\mathrm{d}\,k, n=0,1,...3,$$

(4a)

where

$$f_{02}(z,k)=A_0e^{kz}+B_0e^{-kz}, z\leq 0;$$

$$f_{12}(z,k)=A_1e^{kz}+B_1e^{-kz}, 0\leq z\leq h_1;$$

$$f_{22}(z,k)=\begin{cases}\left(A_2-\dfrac{I}{4\pi\sigma_2 k}e^{-kH}\right)e^{kz}+\\ +\left(B_2+\dfrac{I}{4\pi\sigma_2 k}e^{kH}\right)e^{-kz}, H\leq z<h_2\\ A_2e^{kz}+B_2e^{-kz}, h_1\leq z<H,\end{cases}$$

$$f_{32}(z,k)=\left(A_3e^{kz}+B_3e^{-kz}\right), h_2\leq z\leq\infty.$$

(4b)

and

$$A_0 = \frac{I}{2\pi\sigma_2 k} e^{k2h_1} \frac{\left(e^{k(2h_2-H)} + t_2 e^{kH}\right)(1-t_1)}{e^{2kh_2}\left(e^{k2h_1} - t_1\right) + e^{k2h_1}\left(e^{k2h_1} t_1 - 1\right)t_2},$$

$$B_0 = 0,$$

$$A_1 = B_1 = \frac{I}{4\pi\sigma_2 k} e^{k2h_1} \frac{\left(e^{k(2h_2-H)} + t_2 e^{kH}\right)(1-t_1)}{e^{2kh_2}\left(e^{k2h_1} - t_1\right) + e^{k2h_1}\left(e^{k2h_1} t_1 - 1\right)t_2},$$

$$A_2 = \frac{I}{4\pi\sigma_2 k} \frac{\left[e^{k2h_1} - t_1\right]\left(e^{k(2h_2-H)} + t_2 e^{kH}\right)}{e^{2kh_2}\left(e^{k2h_1} - t_1\right) + e^{k2h_1}\left(e^{k2h_1} t_1 - 1\right)t_2},$$

$$B_2 = \frac{I}{4\pi\sigma_2 k} \frac{e^{k2H_3}\left(e^{k(2h_2-H)} + t_2 e^{kH}\right)\left(1 - e^{k2h_1} t_1\right)}{e^{2kh_2}\left(e^{k2h_1} - t_1\right) + e^{k2h_1}\left(e^{k2h_1} t_1 - 1\right)t_2}, A_3 = 0,$$

$$B_3 = \frac{I}{4\pi\sigma_2 k} \frac{2\left(e^{k(2h_2-H)}\right)\left[e^{k2h_1}\left(1 - e^{k2h_1} t_1\right)\right] + e^{kH}\left[e^{2kh_2}\left(e^{k2h_1} - t_1\right)\right]}{e^{2kh_2}\left(e^{k2h_1} - t_1\right) + e^{k2h_1}\left(e^{k2h_1} t_1 - 1\right)t_2}.$$

(4c)

Similarly, the potential in the n-th layer from the point source placed in the layer with specific conductivity σ_3 (m=3), Fig. 4 is

$$\varphi_{gn3}(r,z) = \int_0^\infty f_{n3}(z,k) J_0(kr) k \, \mathrm{d}k, n = 0,1,...3 \text{ , (5a)}$$

where

$$f_{03}(z,k) = A_0 e^{kz} + B_0 e^{-kz}, z \le 0$$
$$f_{13}(z,k) = A_1 e^{kz} + B_1 e^{-kz}, 0 \le z \le h_1$$
$$f_{23}(z,k) = A_2 e^{kz} + B_2 e^{-kz}, h_1 \le z < h_2$$

$$f_{33}(z,k) = \begin{cases} A_3 e^{kz} + B_3 e^{-kz}, h_2 \le z < H \\ \left(A_3 - \dfrac{I}{4\pi\sigma_3 k} e^{-kH}\right) e^{kz} + \\ + \left(B_3 + \dfrac{I}{4\pi\sigma_3 k} e^{kH}\right) e^{-kz} \quad H \le z \le \infty, \end{cases} \quad (5b)$$

and

$$A_0 = \frac{I}{2\pi\sigma_3 k} \frac{e^{k2h_1}(1-t_1)(1-t_2) e^{k(2h_2-H)}}{\left(e^{k2h_1} - t_1\right)e^{k2h_2} + e^{k2H_1}\left(e^{k2h_1} t_1 - 1\right)t_2}, B_0 = 0$$

$$A_1 = B_1 = \frac{I}{4\pi\sigma_3 k} \frac{e^{k2h_1}(1-t_1)(1-t_2) e^{k(2h_2-H)}}{\left(e^{k2h_1} - t_1\right)e^{k2h_2} + e^{k2H_1}\left(e^{k2h_1} t_1 - 1\right)t_2},$$

$$A_2 = \frac{I}{4\pi\sigma_3 k} \frac{\left(e^{k2h_1} - t_1\right)(1-t_2) e^{k(2h_2-H)}}{\left(e^{k2h_1} - t_1\right)e^{k2h_2} + e^{k2H_1}\left(e^{k2h_1} t_1 - 1\right)t_2},$$

$$B_2 = \frac{I}{4\pi\sigma_3 k} \frac{\left(1 - e^{k2h_1} t_1\right)(1-t_2) e^{k(2h_2+2h_1-H)}}{\left(e^{k2h_1} - t_1\right)e^{k2h_2} + e^{k2H_1}\left(e^{k2h_1} t_1 - 1\right)t_2},$$

$$A_3 = \frac{Ie^{-kH}}{4\pi\sigma_3 k}, B_3 = \frac{Ie^{k(2h_2-H)}}{4\pi\sigma_3 k} \frac{e^{k2h_1}\left(1 - e^{k2h_1} t_1\right) + e^{2kh_2} t_2\left(t_1 - e^{k2h_1}\right)}{\left(e^{k2h_1} - t_1\right)e^{k2h_2} + e^{k2H_1}\left(e^{k2h_1} t_1 - 1\right)t_2}.$$

(5c)

In (4c) and (5c) are $t_1 = (\sigma_1 - \sigma_2)/(\sigma_1 + \sigma_2)$ and $t_2 = (\sigma_2 - \sigma_3)/(\sigma_2 + \sigma_3)$, while other parameters are previously defined and/or can be observed from Figs. 3-4. The Green's function can now be defined as

$$G_{nm}(r,z) = \varphi_{gnm}(r,z)/I, n = 0, 1, 2, 3 \text{ and}$$
$$m = 2, 3 \text{ .} \quad (6)$$

Determination of the system's potential and resistance

The vertical electrode from Fig. 2 is observed. It is assumed that leakage current density per unit length is uniform on the electrode's part in layer 2 ($I_{\text{leak 2}}$) and on the part placed in layer 3 ($I_{\text{leak 3}}$). This approach is justified for quasi-stationary regime. Consequently,

$$I_{\text{leak 2}} = I_2/(h_2 - H_1), I_{\text{leak 3}} = I_3/(H_2 - h_2) \quad (7)$$

where I_2 and I_3 are total currents leaking from the part of the wire placed in layer n=2 (I_2), i.e. n=3 (current I_3). Now, the potential φ_n in layer $n = 0,1,2,3,$ can be expressed as

$$\varphi_n = \int_{H_1}^{h_2} I_{\text{leak 2}} G_{n2}(r,z) \, \mathrm{d}z + \int_{h_2}^{H_2} I_{\text{leak 3}} G_{n3}(r,z) \, \mathrm{d}z \quad (8)$$

Applying the MoM and matching the potential value $\varphi = U$ at the surface point in the middle of the surface of the wire segment placed in the layer 2 defined by the field vector $\vec{R}_2$ and in the middle point of the surface part placed in layer 3 defined by the field vector $\vec{R}_3$,

$$\vec{R}_2 = a\hat{r} + 0.5(H_1 + h_2)\hat{z} = r_2\hat{r} + z_2\hat{z}$$
$$\vec{R}_3 = a\hat{r} + 0.5(H_2 + h_2)\hat{z} = r_3\hat{r} + z_3\hat{z}, \quad (9)$$

it is possible to form linear equation system

$$U = \int_{H_1}^{h_2} I_{\text{leak 2}} G_{22}(r_2, z_2) dz + \int_{h_2}^{H_2} I_{\text{leak 3}} G_{23}(r_2, z_2) dz,$$
$$U = \int_{H_1}^{h_2} I_{\text{leak 2}} G_{32}(r_3, z_3) dz + \int_{h_2}^{H_2} I_{\text{leak 3}} G_{33}(r_3, z_3) dz \quad (10)$$

Solutions of equation systems (11) are currents I_2 and I_3. The resistance of the grounding system is now

$$R_g = U/(I_2 + I_3), \quad (11)$$

OPTIMIZATION METHOD

Grounding system resistance estimation by using analytical expression

Proposed analytical expression for grounding resistance is given as:

$$F_1 = p_0 \cdot H_3^{p_1} + \frac{p_2}{\sigma_1^{p_3}}, F_2 = p_4 \cdot H_3^{p_5} + \frac{p_6}{\sigma_2^{p_7}},$$

$$F_3 = p_8 \cdot H_2^{p_9} + \frac{p_{10}}{\sigma_3^{p_{11}}}, R_e = \left(\frac{1}{F_1} + \frac{1}{F_2} + \frac{1}{F_3}\right)^{-1}$$

(12)

where σ_1-σ_3 and H_2-H_3 are layer specific conductivities and layer highs of three-layer soil model respectively, p_0-p_{11} are constant parameters (coefficients).

The proposed expression is not based on grounding system geometry but represents mathematical description of grounding resistance with respect to layer conductivities and highs.

Parameter determination in the analytical expression

The optimization problem has been defined to find parameter values in the analytical expression. The parameter estimation problem is defined as optimization problem considering the grounding resistance values obtained by using the proposed analytical expression and those calculated by using the more accurate numerical method.

Mathematical description of the optimization problem with objective function (OF) is set as:

$$OF(p,\sigma,H)=\frac{100}{N_d}\cdot\frac{\sum_{i=1}^{N_d}\left|R_c-R_e\right|}{R_c},\text{subject to}:$$

$$R_e>0;\,p_{j,\min}\leq p_j\leq p_{j,\max}$$

(13)

where R_c and R_e are grounding system resistance calculated by using numerical methods and estimated by using the proposed analytical expression according to (12), respectively, and N_d is number of given ground resistance data.

The analytical expression parameters p_0-$p_{11}$are decision variables of the optimization problem (13).

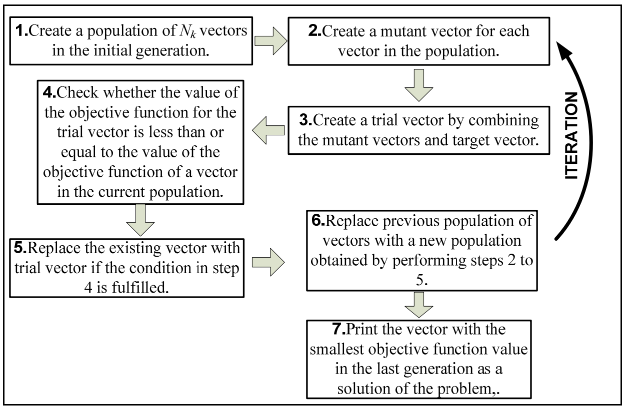

Figure 5. *DE basic procedure.*

The optimization problem (13) is 12-dimensional and nonlinear with constraints. Such type of optimization problem is hard to solve. Because of that, global optimization technique capable to find solution near to global optimum was employed in this case. Differential Evolution (DE) was used here to solve optimization problem (13) and estimate the parameter values in (12). DE [6] belongs to evolutionary optimization class of the global optimization techniques. It is very well described in literature, so it will be very briefly presented here. The basic DE structure is given in Fig. 1DE.

The k-th individual in DE is represented by vector consisting optimization problem decision variables:

$$IN_{DE,k}=[p_{0,k}\;\,p_{1,k}\;\,p_{2,k}\;\,p_{3,k}\;\,p_{4,k}\;\,p_{5,k}\;\,p_{6,k}\;\,p_{7,k}\;\,p_{8,k}\;\,p_{9,k}\;\,p_{10,k}\;\,p_{11,k}]\quad(14)$$

The set of individuals makes population in DE:

$$POP_{DE}=\left[IN_{DE,1}\quad\cdot\quad\cdot\quad\cdot\quad IN_{DE,N_k}\right]^T\qquad(15)$$

DE has two main genetic operator mutation and crossover (recombination) used for making offspring individuals. The mutation generates a mutant individual by adding mutation step to each individual in current population.

The mutant individual (step 2 in Fig. 1DE) is given as [7]:

$$\begin{aligned}IN^g_{DEm,k}&=IN^{g,1}_{DE,k}+F\cdot(IN^{g,2}_{DE,k}-IN^{g,3}_{DE,k}),\\IN^g_{DEm,k}&=[p^g_{m,0,k}\cdots\,p^g_{m,j,k}\cdots\,p^g_{m,11,k}].\end{aligned}\qquad(16)$$

where $IN_{DE,k}^{g,1}$, $IN_{DE,k}^{g,2}$ and $IN_{DE,k}^{g,3}$ are individuals chosen from DE population in g-th generation, F is mutation factor (DE parameter) defined size of the mutation step [6] has value ≤ 1. Individual $IN_{DE,k}^{g,1}$ called base vector and $IN_{DE,k}^{g,2}$ and $IN_{DE,k}^{g,3}$ are so called difference vectors [6]. All these individuals can be chosen in different ways from the current DE population.

The DE crossover operator mixes genes of the mutant and current individual to produce offspring individuals. The offspring individual has specific name in DE called trial individual.

Trial individual (step 3 in Fig. 5) is constructed by choosing parameter value form either an individual in current population or corresponding mutant individual [6]:

$$IN^g_{DEt,k}=\begin{cases}p^g_{m,j,k},\text{if }(\text{rand}_j[0,1]\leq C_r\\p^g_{j,k},\text{otherwise}\end{cases}\qquad(17)$$

DE parameter C_r in (17) is called crossover rate in it is in range (0-1) [6].

Objective function values are calculated for each trial individuals but not for mutant individuals in DE.

After the objective function values for trial individuals are known the selection procedure (steps 4-6 in Fig. 5) in DE is performed. The selection of the individuals in the next DE generation is based on comparison of trial and corresponding current individuals according to [6]:

$$IN^{g+1}_{DE,k}=\begin{cases}IN^g_{DEt,k}\;\text{ if }\;(OF(IN^g_{DEt,k})\leq OF(IN^g_{DE,k}))\\\qquad IN^g_{DE,k}\text{ otherwise}\end{cases}$$

(18)

According to (18) the elitism mechanism is built in DE automatically because if mutation (16) and crossover (17) do not improve current individual it will survive in to the next generation.

NUMERICAL RESULTS

For given set of three-layer soil parameters and grounding resistance data set parameter estimation of the analytical expression (12) was done for $H_1=1\,\text{m}$ and $l=3\,\text{m}$ (Fig. 1). Optimization problem (13) is solved by using existing DE tools in Python programming language environment. The DE tool (scipy.optimize.differential_evolution) built in SciPy Python [7] package is employed to solve the optimization problem and estimate analytical expression parameters. The next DE settings and parameters are used in simulation: "*best1bin*" DE type (strategy), 1000 generation (iteration) number, 25 population size, 0.01 objective function tolerance, [0.5, 1] mutation factor (F) range and 0.7 crossover rate (C_r).

The analytical expression parameter values are estimated by using above procedure for By that, the minimum average error is 7.553838 %.The resistance value R_g obtained by Green's function method and using expression (12) for total 75 data sets is shown in Figure 6.

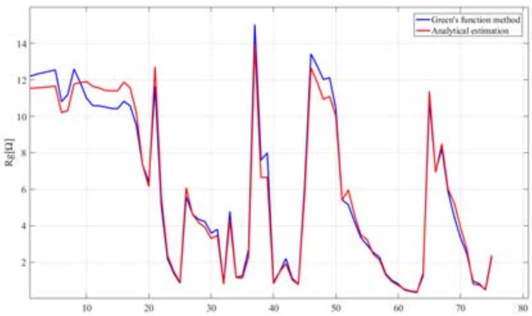

Figure 6. *The resistance value R_g obtained by Green's function method and using expression (12) for total 75 data sets*

CONCLUSION

An optimization procedure for analyzing vertical grounding electrode placed in multilayered soil has been presented in the paper. The procedure is based on the data sets obtained using the approach which includes Green's function and the Method of Moments application. Optimization procedure provides analytical algebraic expression for calculating vertical electrode resistance placed in three-layered soil. The approach is formed in general way and one could expect that it is possible to apply it to other structures of soil (with more homogeneous layers) and for more complex grounding systems. Its accuracy is as better as the values limits of the parameters are better determined

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BIOGRAPHY

Dejan Jovanović was born in Niš, Serbia, in 1989.
He is a PhD student at Faculty of Electronic Engineering, University of Nis, department of Theoretical Electrical Engineering. He received the title of Graduate Engineer at Faculty of Faculty of Electronic Engineering, University of Nis.
His main areas of research include grounding systems and electromagnetic radiations.
He is currently working at Faculty of Electronic Engineering, as a teaching assistant. He teaches the courses: Fundamentals of Electrical Engineering I, Fundamentals of Electrical Engineering II and Electromagnetics.

APROKSIMATIVNA ANALITIČKA FORMULA ZA OTPORNOST VERTIKALNE UZEMLJENE ELEKTRODE POSTAVLJENE U VIŠESLOJNOM MEDIJUMU

Dejan Jovanović, Nenad Cvetković, Miodrag Stojanović, Marinko Barukčić, Željko Hederić, Vladimir Stanković

Rezime: *U radu je izvršena procena i predložen izraz za otpornost uzemljivačke elektrode postavljene u nehomogeno tlo, koje je modelovano kao konačan broj homogenih slojeva konstantne specifične provodnosti.*

Izraz je dobijen optimizacionim postuplkom koji je baziran na obradi seta podataka dobijenih kao rezultat analize koja uključuje korišćenje Grinove funkcije za tačkasti izvor u višeslojnom kao Metod momenata. Pristup je primenjen na vertikalnu uzemljivačku elektrodu postavljenu u troslojnom tlu.

Ključne reči: Grinove funkcije, uzemljenje, metode optimizacije.