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RADIALLY MAGNETIZED RING PERMANENT MAGNET MODELLING IN THE VICINITY OF SOFT MAGNETIC CYLINDER

Abstract: *Radially magnetized ring permanent magnet placed above the soft magnetic cylinder is modelled in this paper using Hybrid Boundary Element Method (HBEM) and a discretization approach based on fictitious magnetic charges. The obtained results are compared to those obtained with Finite Element Method results (FEM).*

Key words: Hybrid boundary element method (HBEM), magnetization charges, magnetic force, permanent magnet (PM).

INTRODUCTION

Since permanent magnets have numerous applications nowadays, the need for the optimization leads to the development of determination methods [1]. The quality of the devices that use permanent magnets greatly depends on the magnet material, magnetization and dimensions. Although there are many different methods one could use in order to analyse permanent magnets, the fact is that Ampere's current model [2] and the Coulombian approach [3] are the ones most frequently used. Up to now, the authors have proposed complex and simplified solutions for the force between cylindrical magnets [4], axially [3], [5] and radially [6], [7] magnetized ring permanent magnets or cuboidal magnets [8]. Also, there are many papers where the magnetic force between permanent magnet and magnetic plate is modelled too, but it is performed only using image method [2], [9].

The hybrid boundary element method (HBEM) [10], developed at the Faculty of Electronic Engineering of Niš, is used for calculating the magnetic force exerts between radially magnetized ring permanent magnet and magnetic cylinder. Until now, it has been applied for solving multilayered electromagnetic problems, microstrip lines [11], or electromagnetic field determination in the vicinity of cable terminations [10]. Up to now, the method has been applied for solving 2D and 3D electromagnetic problems.

THEORETICAL BACKGROUND

Along with HBEM, the approach that is used for solving the force between ring permanent magnet (PM) and cylinder made of magnetic material is based on fictitious magnetization charges and discretization technique [5], [7]. The simplest way to calculate the interaction magnetic force is to discretize a permanent magnet into system of circular loops loaded with magnetic charges. Using results for interaction magnetic force between two circular loops [7], the force that exerts between ring magnet and magnetic cylinder, can be obtained. Summing the contribution of PM and cylinder segments (or both PMs in second considered configuration) by using uniform discretization technique will give the force expression. This approach enables rapid magnetic force calculation and force analyses within the parameters.

SEMI-ANALITICAL APPROACH FOR MAGNETIC FORCE CALCULATION

Radially magnetized ring PM, placed above the cylinder made of linear magnetic material with relative permeability μ_{r2} , is taken under consideration (Figure 1).

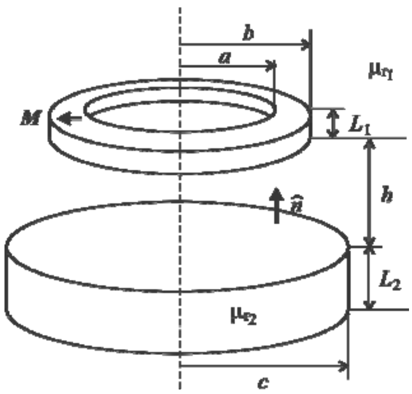

Figure 1. *Ring permanent magnet above magnetic cylinder*

Since the magnet is magnetized in radial direction, it is obvious that the fictitious magnetic charge density is composed of volume, $\rho_m = -\nabla \cdot \boldsymbol{M} = -M/r$, and surface charge density, $\eta_m = \hat{n} \cdot \boldsymbol{M}$. Therefore, the volume, inner and outer cover of ring permanent magnet can be consider as a system of circular loops uniformly loaded with magnetic charges.

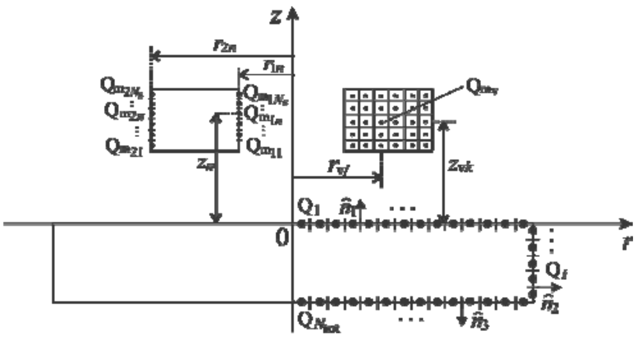

Figure 2. *Discretization model*

Taking in to account that the cylinder is made of linear magnetic material with permeability μ_{r2}, the influence of the magnetic material can be replaced with the system of toroidal magnetic sources that exist on the boundary surface of two different materials (on cylinder cover and bases).

Discretization model and distribution of magnetic charges as well as toroidal sources are shown in the Figure 2. Magnetic scalar potential of the considered system becomes.

$$\begin{aligned}\varphi_m = \frac{1}{2\pi^2}\Big(&S(Q_{m1}, r, r_{1n}, z, z_n, n, N_s) + \\ &+ S(Q_{m2}, r, r_{2n}, z, z_n, n, N_s) \\ &+ \sum_{k=1}^{N_{v1}} S(Q_{mv}, r, r_{vj}, z, z_{vk}, j, N_{v2}) \\ &+ S(Q_i, r, r_i, z, z_i, i, N_{tot})\Big).\end{aligned} \quad (1)$$

where

$$S(Q, r, a, z, b, n, N) = \sum_{n=1}^{N} Q \frac{K(r, a, z, b)}{\sqrt{(r+a)^2 + (z-b)^2}},$$

$$K(r, a, z, b) = K\left(\frac{\pi}{2}, \frac{4ra}{(r+a)^2 + (z-b)^2}\right).$$

Considering the Figure 2, following parameters of PM covers can be expressed as

$$z_n = h + \frac{2n-1}{2N_s} L_1,\ r_{1n} = a,\ r_{2n} = c,\ n = 1, 2, \ldots, N_s, \quad (2)$$

while the magnetic charges of inner and outer covers segments are

$$Q_{m1\,n} = -M 2\pi r_{1n} \frac{L_1}{N_s},$$

$$Q_{m2\,n} = M 2\pi r_{2n} \frac{L_1}{N_s},\ n = 1, 2, ..., N_s. \quad (3)$$

where N_s is the number of segments of each PM cover.

The volume segments parameters are

$$z_{vk} = h + \frac{2k-1}{2N_{v1}} L_1,\ k = 1, 2, \ldots, N_{v1}, \quad (4)$$

$$r_{vj} = a + \frac{2j-1}{2N_{v2}}(c-a),\ j = 1, 2, \ldots, N_{v2}, \quad (5)$$

and their respective magnetic charges:

$$Q_{mv} = Q_{mvm} = -M 2\pi L_1 \frac{c-a}{N_{v1}N_{v2}},\ m = 1, 2, ..., N_v, \quad (6)$$

$N_v = N_{v1} \cdot N_{v2}$ is the total number of volume PM segments. Positions of toroidal magnetic sources on the cylinder cover and bases are (r_i, z_i). For the upper base it is

$r_i = \frac{2i-1}{2N_1} b,\ z_i = 0,\ i = 1, 2, \ldots, N_1$, while the radius of toroidal sources is $a_{e1} = \Delta r_1/\pi,\ \Delta r_1 = b/N_1$.

Positions of the cover sources are

$$r_i = b,\ z_i = \frac{2N_1 - 2i + 1}{2N_2} L_2,\ i = N_1 + 1, \ldots, N_1 + N_2,$$

with radius $a_{e2} = \Delta z_2/\pi,\ \Delta z_2 = L_2/N_2$ and for the lower bases toroidal sources the following relations are satisfied

$$r_i = \frac{2i - 2N_1 - 2N_2 - 1}{2N_1} b,\ z_i = -L_2,\ i = N_1 + N_2, \ldots, 2N_1 + N_2$$

and $a_{e3} = a_{e1}$.

N_1 is the number of magnetic sources on each cylinder base, N_2 is the number of magnetic sources on the cylinder cover, $N_{tot} = 2N_1 + N_2$ is the total number of cylinder magnetic sources.

Magnetic field strength vector is $\boldsymbol{H} = -\mathrm{grad}(\varphi_m)$.

The normal component of magnetic field vector and cylinder surface charges completes the following relation

$$\hat{\boldsymbol{n}}_k \cdot \boldsymbol{H}_k^{(0+)} = \frac{-\mu_{r2}}{\mu_{r1} - \mu_{r2}} \eta_{mi},$$

$$\eta_{mi} = \frac{Q_i}{2r_i\pi\Delta r_i},\ i = 1, 2, \ldots, N_{tot},\ k = 1, 2, 3., \quad (7)$$

where $\hat{\boldsymbol{n}}_k$ is the unit vector of outgoing normal, ($\hat{\boldsymbol{n}}_1 = \hat{\boldsymbol{z}}$, $\hat{\boldsymbol{n}}_2 = \hat{\boldsymbol{r}}$, $\hat{\boldsymbol{n}}_3 = -\hat{\boldsymbol{z}}$). Using the point matching method for the normal magnetic field component, the system of linear equations is formed. The solution of this system gives the values of unknown charges of toroidal sources, Q_i.

The distribution of normalized magnetic sources, $Q_{i\,\mathrm{nor}} = Q_i / Mh^2$, along the boundary surface of two different magnetic materials is presented in Figure3, for the system parameters: $a/L_1 = 3.0$, $b/L_1 = 1.0$, $c/L_1 = 4.0$, $L_2/L_1 = 0.5$, $h/L_1 = 1.0$, $\mu_{r1} = 1$, $\mu_{r2} = 3$, $N_s = 100$, $N_v = 100$, $N_p = 200$.

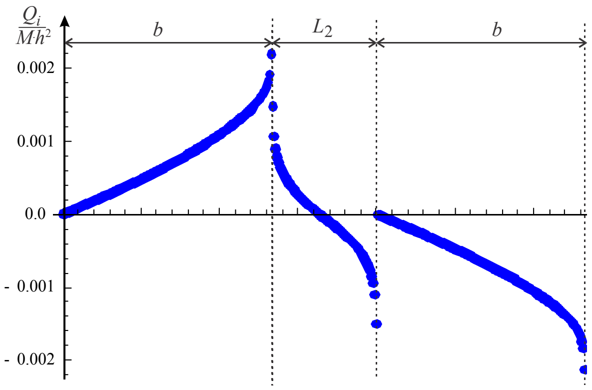

Figure 3. *Distribution of magnetic sources along the boundary surface of two different magnetic materials*

After that, the magnetic scalar potential of magnetic cylinder can be calculated using the expression

$$\varphi_{mc} = \frac{1}{2\pi^2} S(Q_i, r, r_i, z, z_i, i, N_{tot}), \qquad (8)$$

along with magnetic field and magnetic flux density vector anywhere in the cylinder vicinity. Final step would be superposition of contributions for all PM segments and toroidal magnetic sources. This would give us the expression for the force that exerts between PM and cylinder made of linear magnetic material placed in the environment of permeability μ_0:

$$F_z = \sum_{n=1}^{N_s} \sum_{i=1}^{N_{tot}} \Big(F_{z_p}(Q_{m1n}, Q_i, r_i, r_{1n}, z_i, z_n) + F_{z_p}(Q_{m2n}, Q_i, r_i, r_{2n}, z_i, z_n) \Big) + \sum_{k=1}^{N_{v1}} \sum_{j=1}^{N_{v1}} \sum_{i=1}^{N_{tot}} \Big(F_{z_p}(Q_{mv}, Q_i, r_i, r_{vj}, z_i, z_{vk}) \Big), \qquad (9)$$

where the force between two circular loops is

$$F_z = \mu_0 \frac{Q_{m1} Q_{m2}}{2\pi^2} \times \frac{(z_m - z_0) E\left(\frac{\pi}{2}, \frac{4 r_0 r_m}{(r_m + r_0)^2 + (z_m - z_0)^2}\right)}{\left((r_m - r_0)^2 + (z_m - z_0)^2\right)\sqrt{(r_m + r_0)^2 + (z_m - z_0)^2}} = F_{z_p}(Q_{m1}, Q_{m2}, r_0, r_m, z_0, z_m).$$

(10)

NUMERICAL RESULTS

It has already been proven that it is enough to take $N_s = 100$ surface segments and $N_v = 100$ volume segments for PM to achieve high accuracy [7]. In order to determine the number of toroidal sources the convergence of the results need to be tested. The normalized intensity of magnetic force, $F_z^{\mathrm{nor}} = F_z / \mu_0 M^2 L_1^2$, for different number of toroidal sources is calculated for parameters: $\mu_{r1} = 1$, $\mu_{r2} = 4$, $a/L_1 = 1.0$, $c/L_1 = 3.0$, $b/L_1 = 3.0$, $L_2/L_1 = 0.5$, $h/L_1 = 0.5$. Considering the values in the Table I it is obvious that the results of approach used are confirmed with Finite element method results (FEM) [13]. In order to reduce the calculation time, number of toroidal sources is limited to $N_{tot} = 371$. In that case the relative error is less than 0.2% .

Table 1. *Convergence of the results*

N_{tot}	F_z^{nor}	F_z^{nor} (FEM)	Relative error [%]
93	-0.583406	-0.586005	0.44
186	-0.584359		0.28
371	-0.584861		0.19
557	-0.585028		0.17
743	-0.585115		0.15
929	-0.585170		0.14

Normalized axial force, $F_z^{\mathrm{nor}} = F_z / \mu_0 M^2 L_1^2$, between ring magnet and magnetic cylinder versus normalized axial displacement of PM, h/L_1, for variable PM radius is presented in the Figure 4. It is shown for configuration parameters $c/L_1 - a/L_1 = 1.0$, $b/L_1 = 3.0$, $L_2/L_1 = 0.5$, $\mu_{r1} = 1$, $\mu_{r2} = 4$.

Figure 5 shows normalized axial magnetic force versus cylinder permeability, μ_{r2}, for different normalized distance between PM ring and cylinder, h/L_1 when it is $a/L_1 = 1.0$, $b/L_1 = 3.0$, $c/L_1 = 3.0$, $L_2/L_1 = 0.5$, $\mu_{r1} = 1$.

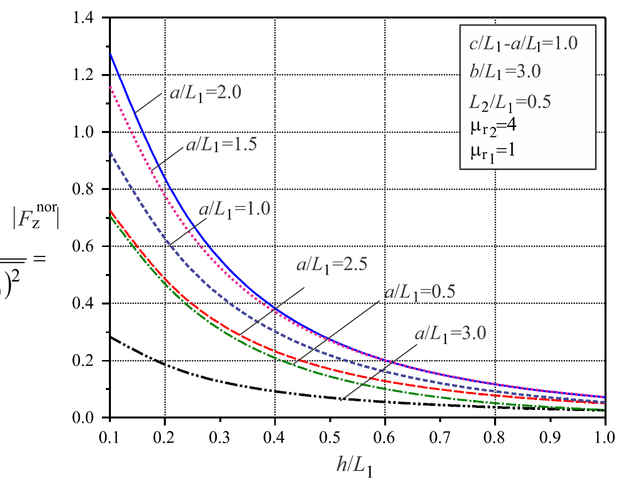

Figure 4. *Normalized axial magnetic force versus ratio h/L_1 for different normalized PM radius, a/L_1.*

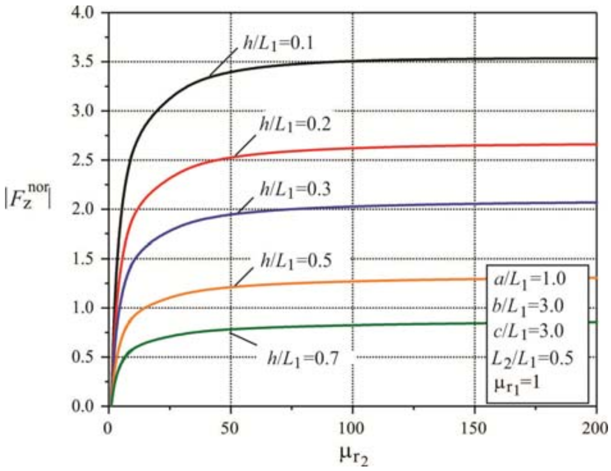

Figure 5. *Normalized axial magnetic force versus cylinder permeability, μ_{r2}.*

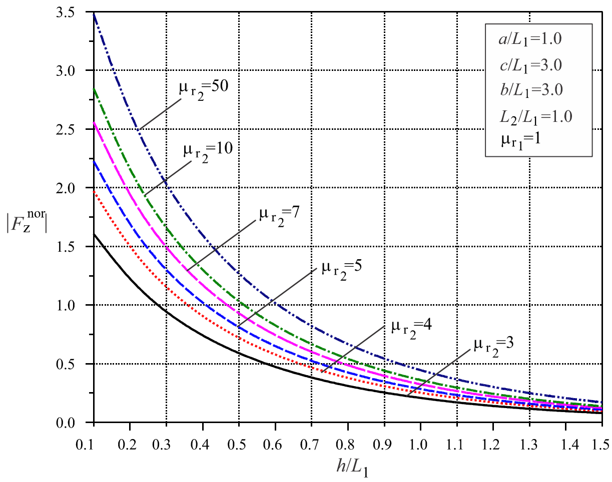

Figure 6. *Axial magnetic force versus ratio h/L_1 for different cylinder permeability, μ_{r2}*

Normalized axial force, that between permanent magnet and soft magnetic cylinder, versus normalized distance between permanent magnet and cylinder, h/L_1, for variable relative permeability of magnetic cylinder is shown in the Figure 6, for for the system parameters $a/L_1 = 1.0$, $b/L_1 = 3.0$, $c/L_1 = 3.0$, $L_2/L_1 = 1.0$, $\mu_{r1} = 1$.

Figure 7 presents magnetic force, versus cylinder permeability, μ_{r2}, for different height ratio of permanent magnet and magnetic cylinder, L_2/L_1, when it comes to permanent magnet parameters: $a/L_1 = 1.0$, $b/L_1 = 3.0$, $c/L_1 = 2.0$, $h/L_1 = 0.5$, $\mu_{r1} = 1$.

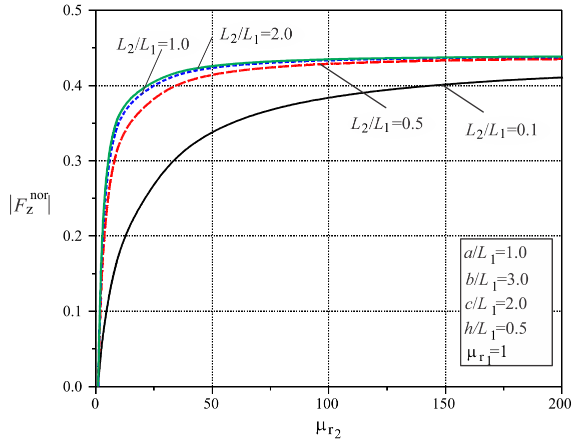

Figure 7. *Axial magnetic force versus cylinder permeability μ_{r2} for different height ratio L_2/L_1*

CONCLUSION

Determination of magnetic force between ring PM placed above cylinder made of linear magnetic material is presented. It is performed using semi-analytical approach based on magnetization charges and HBEM. An algorithm presented is easily implemented in any standard software development environment and it enables rapid parametric studies of the interaction force. The results of the presented approach have been successfully confirmed by FEM results. Axial forces calculation using presented approach for mentioned parameters is performed with Intel Core i5-2320 CPU Quad Core at 3GHz and 8GB RAM memory, 64-bit operating system and it took around 0.76 seconds of run time for one calculation. Interaction forces are also determined on the same computer using FEMM 4.2 software and the execution time was 5.6 minutes for about 2 million finite elements. The advantage of this approach is time efficiency and possibility of solving a configuration that contains an object made of magnetic material with finite dimensions, while the majority of published papers deals with interaction force between PM and infinite magnetic plain.

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BIOGRAPHY

Ana Vučković was born in Niš, Serbia, in 1977. She holds the PhD in the field of Electrical Engineering and Computing. PhD studies she finished at the Faculty of Electronic Engineering, University of Niš, Serbia.

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MODELIRANJE RADIJALNO MAGNETIZOVANOG PRSTENASTOG MAGNETA U BLIZINI MEKOG MAGNETNOG CILINDRA

Ana Vučković, Nebojša Raičević, Mirjana Perić

Rezime: *U radu je prikazano modeliranje radijalno magnetizovanog prstenastog trajnog magneta koji je postavljen iznad mekog magnetnog cilindra uz pomoć metode hibridnog graničnog elementa (eng. HBEM -Hybrid Boundary Element Method) i metode diskretizacije bazirane na fiktivnim magnetnim naelektrisanjima. Dobijeni rezultati su upoređeni sa rezultatima metode konačnih elemenata (eng. FEM - Finite Element Method).*

Ključne reči: metoda hibridnog graničnog elementa (HBEM), magnetizacija, magnetna sila, trajni magnet.