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RELIABILITY CALCULATION AND MONTE CARLO METHOD

Abstract: *A simple, effective and easy to implement method for the reliability calculation of the discrete systems with randomly chosen parameters is presented in this paper. Since there are many reasons for reliability decreasing, the influence of system instability to the system reliability is considered in this paper. This method is very applicable in practice, for example in safety engineering, failure analysis, etc. Random choice of parameter value for which the system has the largest probability of stability enables the maximum reliability. In this way many problems can be avoided, for example failures of components and system. The proposed method is illustrated on an example of discrete band frequency damper and its validity has been confirmed by Monte Carlo method.*

Key words: Reliability, Probability of stability, Monte Carlo method

INTRODUCTION

Reliability analysis is a topic with applications in so many fields, for example safety engineering, failure analysis, etc. One of the purposes of system reliability analysis is to identify the weakness in a system and to quantify the impact of component failures. With modern technology and higher reliability requirements systems become more complicated. The complex systems, such as chemical plants, transportation system, manufacturing plants and complex technological systems should have high reliability considering the consequences of their failures and the impact they have on further operations and people security. Reliability is one of the quality characteristics that it is required from the system [1]. Reliability engineering deals with estimation, prevention and management of engineering uncertainty and risk of failure.

The reliability $R(t)$ is usually defined as probability that a component or a system will operate properly for a specified period of time under the design operating conditions without filure. The reliability theory is based on the application of probability theory since the moments of failures are random variables. The reliability is defined by the next relation:

$$R(t) = \int_{t_1}^{\infty} p(t)dt$$

where $p(t)$ is probability density distribution. $R(t)$ presents the probability that the system in the observed moment will work correctly.

There are many reasons for reliability decreasing. In this paper, the influence of system instability to the system reliability is considered. It is assumed that the system is reliable as long as is stable. The method for the probability of stability estimation can be used for the system reliability analysis, [2-4]. System reliability is in correlation with probability stability which is verified using scatter diagram, [5]. If parameters of the system have stationary values in certain time period, then the system is reliable in that period of time. Random choice of parameter value for which the system has the largest probability of stability enables the maximum reliability. In this way, many problems can be avoided, for example failures of components and system.

However, parameter values are usually time dependent random variables. During the exploitation, the system is subjected to different factors influencing its work. These factors can change the values of system parameters and its reliability and in the limit case can bring system to the instability or failure, [6-8].

This method is applicable for the enhancement of the safety and reliability of many types of systems. Since reliability and safety deal with uncertainty, Monte Carlo method can be used.

In this paper Monte Carlo method, [9], was used to verify the validity of the results obtained by the proposed method. Monte Carlo method can be defined as statistical method, where statistical simulation is defined to be any method that utilizes sequences of random numbers to perform the simulation. Monte Carlo method gives approximate solution of different types of problems by performing statistical sampling experiments. This method can provide an approximate solution quickly and with the high level of accuracy, because the more simulation is performed, the more accurate approximation is obtained.

The application of the Monte Carlo method does not require integration by the stability region. The reliability is calculated as the quotient of the number of samples that belong to the stability region and the number of all scanned samples. For the samples we consider values of system parameters. Random parameters are described in terms of their bounds.

The Monte Carlo method gives almost identical results as the method for the reliability calculation. The experiments were performed for the second and the third order systems with normal probability distribution of parameters. The random number generator was used to generate the values of the parameters with normal distribution. The experiment was performed on the 100.000 samples. The results are given in tables.

Presented method is illustrated by the example of the second order discrete system, i.e., discrete band frequency damper

THE PROBABILITY OF STABILITY CALCULATION

The mathematical model of the n[th] order discrete system is given by the following difference equation:

$$\sum_{i=0}^{n} a_i x(kT+n-i)=0,\ T=1,\ a_0=1 \qquad (1)$$

where parameters a_i are random variables with probability distribution densities $p_i(a_i)$, and T is sampling time.

The stability region of the system (1) in the parametric space is determined firstly. The system stability is determined from the next equation:

$$z^n + a_1 z^{n-1} + \cdots + a_n = 0 \qquad (2)\backslash$$

The necessary and sufficient condition for the system (1) stability is that all zeros of its characteristic polynomial should be located inside the unit circle in the z-plane.

The bilinear transformation method is used to test this condition, having the inside of the unit circle mapped into the left half of the complex plane. A new equation is obtained:

$$s^n + \varphi_{n-1} s^{n-1} + \cdots + \varphi_0 = 0 \qquad (3)$$

with coefficients $\varphi_i(a_1, a_2, \ldots, a_n) = \varphi_i$. Applying the Hurwitz criterion, the stability region S_n of difference equation (1) is obtained. The necessary and sufficient condition, which requires all zeroes of the equation (3) to be located on the left half of the complex plane, is that all diagonal minors D_i of the Hurwitz matrix D are greater than zero. The stability region S_n is determined from the following relations:

$$D_1 = \varphi_{n-1} > 0,\quad D_2 = \begin{vmatrix} \varphi_{n-1} & \varphi_{n-3} \\ \varphi_n & \varphi_{n-2} \end{vmatrix} > 0, \text{ etc.}$$

For the first order system the stability region, S_1, is a range defined as: $-1 < a_1 < 1$.

For the second order system the stability region, S_2, is given by the next set of inequalities:

$$\begin{aligned} &1 - a_1 + a_2 \geq 0 \\ &1 + a_1 + a_2 \geq 0 \\ &a_2 \leq 1 \end{aligned} \qquad (4)$$

The stability region S_2 is presented in Fig. 1, where N_2 presents the region of instability. In the case of the second order system, the stability region is a triangle.

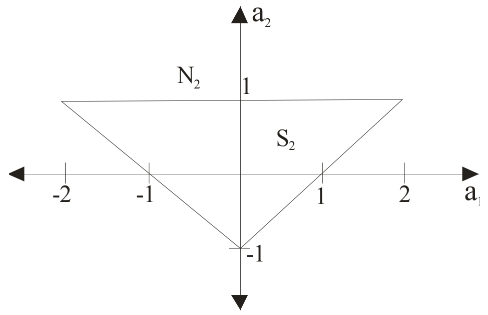

FIGURE 1. *The stability region, S_2, of the second order discrete system*

For the third order system, the stability region, S_3, is given by the next set of inequalities:

$$\begin{aligned} &a_1 + a_2 + a_3 > -1 \\ &a_1 - a_2 + a_3 < 1 \\ &a_1 a_3 + 1 > a_2 + a_3^2 \end{aligned} \qquad (5)$$

The stability region is shown on Fig. 2.

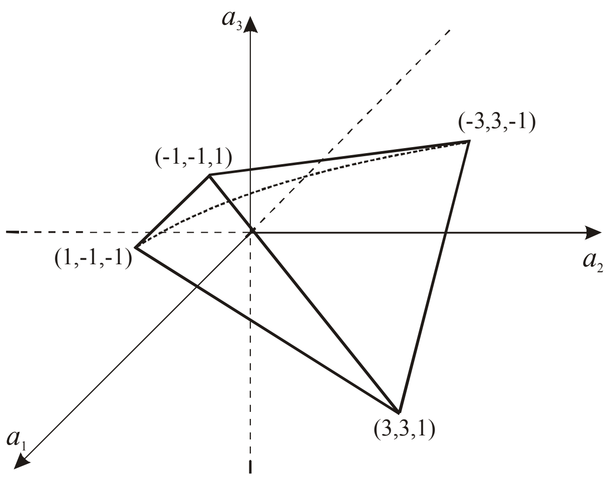

FIGURE 2. *The stability region, S_3, of the third order discrete system*

If the probabilities are given, then the total distribution density, for the independent parameters, is given by:

$$p(a_1, \ldots, a_n) = \prod_{i=1}^{n} p_i(a_i) \qquad (6)$$

The probability of stability of the difference equation (1) is:

$$P = \int_{S_n} \cdots \int p(a_1, \ldots, a_n)\, da_1 \cdots da_n \qquad (7)$$

In the case when the stability region is determined by more than three parameters, the problem becomes more complex. Particularly, surfaces limiting the stability region are usually defined by very complex mathematical relations. It is also necessary to integrate by the region S_n, (7), which is very complicated considering the complex distribution densities of the random parameters.

For higher order systems it is very important to estimate the probability of stability for the practical applications. For the probability of stability estimation two theorems can be applied very effectively and they are given in [2].

Method for the probability of stability estimation can be used for reliability analysis. There are many reasons for reliability decrease; however, we here considered the case when system instability is the cause. It is assumed that the system is reliable as long as it is stable.

System is reliable if it is situated in the stability region, so it is necessary to determinate the conditions that will enable the system to remain stable during certain time interval. In order to find adequate probability density function of random variable, which is in our case the value of system parameter, we have used the example of one dimensional Brownian motion of particles. It can be proved that random variable has approximately $N(0;\sigma^2 t)$ probability distribution, [9]. For the second and the third order discrete systems it is possible to calculate reliability exactly.

For the higher order discrete systems, it is difficult to calculate reliability due to complexity of stability region. In that case, reliability must be estimated using the method for the probability of stability estimation. This estimation has significant value in practical applications.

THE VERIFICATION OF RESULTS USING MONTE CARLO METHOD

In this paper, the Monte Carlo method is used to confirm the results obtained by the method for the reliability estimation. The Monte Carlo method gives almost identical results like the method for the reliability estimation. The results are presented in the tables for the second and the third order systems with normal distribution of parameters. Reliability, P_{S_n} is obtained by the method for the probability stability estimation and the system reliability, P, is calculated using the Monte Carlo method. The reliabilities P_{S_n} and P have almost identical values. The random number generator was used to generate parameter values with normal distribution, having the sample size 1000000.

The calculation of the reliability using Monte Carlo method is much easier because there is no need for integration by the region of stability, only the limits of stability region are required. The reliability is calculated as the quotient of the number of samples that belong to the region of stability and the number of all scanned samples. We consider the values of system parameters using the samples. For the higher order systems, Monte Carlo method provides very good results.

Table 1. *Results for the second order system with normal distribution of parameters*

σ_1	0.1	0.1	0.1	0.3	0.7
σ_2	0.1	0.2	0.3	0.3	0.8
P	1	0.82	0.66	0.471	0.124
P_{S_2}	0.974	0.814	0.634	0.452	0.104

Table 2. *Results for the third order system with normal distribution of parameters*

σ_1	0.7	0.3	0.2	0.1	0.1
σ_2	0.8	0.4	0.3	0.2	0.1
σ_3	0.9	0.5	0.4	0.2	0.1
P	0.023	0.161	0.223	0.57	0.932
P_{S_3}	0.019	0.115	0.214	0.56	0.921

The presented method is illustrated wit the example of the second order discrete system (discrete band frequency damper) shown in Fig. 3. This example shows the practical application of the method. The resistances have the constant values, whereas the capacitance and resistance change their values by the normal probability distribution, $N(0;\sigma^2 t)$. T is the sampling time.

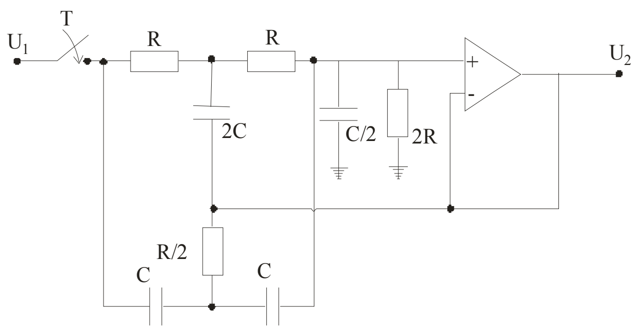

FIGURE 3. *The discrete band frequency damper*

The characteristic polynomial of the system is:

$$z^2 + a_1 z + a_2 = 0 \quad (8)$$

where a_1 and a_2 are functions of the capacitance C and the resistance R.

Using the bilinear transformation, the new equation is obtained:

$$s^2 3C^2 R^2 + s2CR + 3 = 0 \quad (9)$$

Using the presented method for $\overline{C} = 0$, $\sigma_1 = 0.7$, $\overline{R} = 0$, $\sigma_2 = 0.8$, the reliability of the system is R=0.104. The other values of parameters can be selected from the Table 1.

CONCLUSION

There are many reasons for reliability decreasing. In this paper, the influence of system instability to the system reliability has been considered. It is assumed that the system is reliable as long as is stable. Due to its simplicity, the method for the probability of stability estimation can be used to perform the system reliability analysis. The selection of adequate values of parameters provides the maximum reliability. Thus many problems can be avoided in the field of system safety, failure of the components and systems. The validity of the proposed method is approved by the well - known Monte Carlo method. The experiments for the second and the third order systems were performed with normal distribution of parameters, giving almost identical results as the Monte Carlo method.

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BIOGRAPHY

Biljana Samardzic graduated from the Faculty of Electronic Engineering in Nis. She received the M.Sc. and PhD degrees from the Faculty of Electronic Engineering, too. Her research interests are bond graph modeling and simulation, chaos modeling and parameter estimation.

IZRAČUNAVANJE POUZDANOSTI I MONTE KARLO METOD

Biljana Samardžić, Bojana Zlatković

Rezime: *Jednostavan, efikasan, lako primenljiv metod za izračunavanje pouzdanosti kod diskretnih sistema sa slučajno izabranim parametrima je predstavljen u ovom radu. Pošto postoji mnogo razloga za smanjenje pouzdanosti, u ovom radu se razmatra uticaj nestabilnosti sistema na pouzdanost. Ovaj metod je veoma primenljiv u praksi, npr. u inžinjerstvu zaštite, analizi otkaza itd. Slučajan izbor vrednosti parametara za koje sistem ima najveću verovatnoću stabilnosti omogućava maksimalnu pouzdanost. Na ovaj način mogu da se izbegnu mnogi problemi, kao što je, na primer, otkaz komponenti i sistema. Predloženi metod je ilustrovan na primeru diskretnog prigušivača opsega frekvencija, a njegova ispravnost je potvrđena Monte Karlo metodom.*

Ključne reči: pouzdanost, verovatnoća stabilnosti, Monte Karlo metod.