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APPLICATION OF LAPLACE APPROXIMATION FORMULA IN PERFORMANCE ANALYSIS OF WIRELESS RELAY COMMUNICATION SYSTEM IN MULTIPATH FADING ENVIRONMENT

Abstract: *In this paper, Laplace approximation formula is efficiently used to calculate infinite-series expression for average level crossing rate (LCR) of the product of Nakagami-m and kappa-mu random processes. The results can be used in performance analysis of dual-hop relay wireless mobile communication system in specific multipath fading environment when the first section is under the influence of NLOS multipath environment while the second section is under the influence of LOS multipath fading environment. The influences of multipath fading parameters on the LCR of the proposed model are examined, graphically presented and discussed. Moreover, analytical approach of applying Laplace approximation formula in multi-hop systems is further considered by obtaining closed form expressions for the LCR of the product of three and four Nakagami-m processes, respectively.*

Key words: average level crossing rate, kappa-mu multipath fading, Laplace approximation formula, Nakagami-m distribution, relay communications system.

INTRODUCTION

Relay communications provide efficient and reliable transmission technique where several relay terminals function between transmitter and receiver of wireless communication system enabling wide signal area coverage [1, 2]. Secure transmission as well as less system power consumption can be successfully achieved [3]. Moreover, optimal energy consumption of wireless transmission systems is significant from the standpoint of reducing electromagnetic pollution [4]. In environments susceptible to earthquakes, floods etc. improvement of wireless system performances using reliable transmission techniques can be of crucial importance [5]. Further, multi-hop amplify-and-forward (AF) relay system signal envelope can be modeled as the product of two or more random variables where the first random variable is the signal envelope at the first section, the second variable is the signal envelope at the second section while the last random variable is the signal envelope at the last section [6, 7]. Multi-hop relay systems can be applied in different scenario to increase safety such as: car-to-car (C2C) and car-to-everything (C2X) communications, and to enable reliable connections via drones [6]. Moreover, amplify-and-forward (AF) multi-hop relays beside its reliability and efficiency are simple for practical realization since they don't require decoding and other signal processing. Accordingly, the AF multi-hop relays can be used in environments to increase overall safety. Moreover, second order metrics such as: LCR and AFD take into account rapidly time varying channels which are crucial to investigate in C2C communications, C2X communications and communications via drones to increase safety

In paper [8], Laplace approximation formula was efficiently applied for obtaining closed form solution for average level crossing rate (LCR) of the product of *N* Rayleigh random processes. Moreover, this expression is used for calculation of average fade duration (AFD) of wireless relay communication system with two, three or more sections operating over multipath fading channel. LCR and AFD of the product of two Nakagami-*m* random variables are calculated in paper [9].

The second order statistical measure such as LCR determines the average number that the signal envelope crosses threshold in one direction. LCR can be obtained as average value of the first derivative of the random process. Moreover, LCR can be used for obtaining AFD which determines the meantime that the signal envelope is below threshold and can be evaluated as the ratio of the outage probability (OP) and LCR. On the other hand, OP is the first order statistical measure which determines the probability that the signal envelope average value is below threshold and can be calculated using cumulative distribution function (CDF).

Nakagami-*m* distribution can be used to describe small scale signal envelope variation in multipath, non line-of-sight (NLOS) fading channel with more than one propagation cluster [10]. Moreover, Nakagami-*m* distribution for m=1 approximate Rayleigh conditions.

On the other hand, kappa-mu (k-μ) distribution is general distribution which means that for different values of kappa and mu parameters other well-known distributions such as Rayleigh, Rice and Nakagami-m can be approximated. The kappa-mu distribution can describe small scale variation in line-of-sight (LOS) fading environment. The kappa parameter, also known as Rice factor can be calculated as the ratio of dominant components power and scattering components power while mu parameter known as fading severity, determines the number of propagation clusters [11, 12, 13].

In this paper, Laplace approximation formula is applied for calculation of infinite-series expression of the LCR of the product of Nakagami-m and kappa-mu random processes. Numerical results can be used in performance analysis of relay wireless communication system operating over dual-hop multipath fading channel including the scenario when the first section is under the influence of NLOS environment and the second section is under the influence of LOS environment. Moreover, the closed form expression for LCR of the product of three and four Nakagami-m random processes using Laplace approximation formula for two-folded and three-folded integrals is obtained.

LCR OF TWO RANDOM PROCESSES

The product of Nakagami-m random process and kappa-mu random process is:

$$z = xy, \tag{1}$$

where x follows Nakagami-m distribution and y follows kappa-mu distribution, respectively [10]:

$$p_x(x) = \frac{2}{\Gamma(m_1)}\left(\frac{m_1}{\Omega_1}\right)^{m_1} x^{2m_1-1} e^{-\frac{m_1}{\Omega_1}x^2}, \quad x \geq 0, \tag{2}$$

where m_1 is Nakagami-m fading severity parameter, $\mathbf{\Omega_1}$ is mean power of Nakagami-m random variable,

$$p_y(y) = \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu} \Omega_2^{\frac{\mu+1}{2}}} y^{\mu} I_{\mu-1}\left(2\mu\sqrt{\frac{k(k+1)}{\Omega_2}}y\right) e^{-\frac{\mu(k+1)}{\Omega_2}y^2}, \tag{3}$$

where k is Rice factor, μ is fading severity and $\mathbf{\Omega_2}$ is mean power of kappa-mu random variable, $I_v(\cdot)$ present modified Bessel function of the first kind order v. The first derivative of the product of Nakagami-m and kappa-mu random process is:

$$\dot{z} = \dot{x}y + x\dot{y}. \tag{4}$$

Random process $\dot{z}$ has conditional Gaussian distribution since $\dot{x}$ and $\dot{y}$ are Gaussian random processes. The variance of the first derivative of z is [14]:

$$\sigma_{\dot{z}}^2 = y^2\sigma_{\dot{x}}^2 + x^2\sigma_{\dot{y}}^2, \tag{5}$$

where,

$$\sigma_{\dot{x}}^2 = \pi^2 f_m{}^2 \frac{\Omega_1}{m_1}, \tag{6}$$

$$\sigma_{\dot{y}}^2 = \pi^2 f_m{}^2 \frac{\Omega_2}{\mu(k+1)}. \tag{7}$$

After substitution (6) and (7) in (5), expression for variance of $\dot{z}$ is:

$$\sigma_{\dot{z}}^2 = \frac{4\pi^2 f_m{}^2 \Omega_1}{m_1} y^2 \left(1 + \frac{z^2}{y^4}\frac{\Omega_2 m_1}{\mu(k+1)\Omega_1}\right). \tag{8}$$

Joint probability density function (JPDF) of $z, \dot{z}$ and y is:

$$\begin{aligned} p_{z,\dot{z},y}(z,\dot{z},y) &= p_{\dot{z}|yz}(\dot{z} \mid yz)\, p_{yz}(yz) = \\ &= p_{\dot{z}|yz}(\dot{z} \mid yz)\, p_y(y)\, p_z(z \mid y). \end{aligned} \tag{9}$$

Conditional probability density function of z is:

$$p_z(z \mid y) = \left|\frac{dx}{dz}\right| p_x\left(\frac{z}{y}\right), \tag{10}$$

where,

$$\frac{dx}{dz} = \frac{1}{y}. \tag{11}$$

After substitution (10) in (9), the expression for JPDF becomes:

$$p_{z,\dot{z},y}(z,\dot{z},y) = \frac{1}{y} p_{\dot{z}|yz}(\dot{z} \mid yz)\, p_y(y)\, p_x\left(\frac{z}{y}\right). \tag{12}$$

JPDF of z and derivative of z can be obtained by integrating of (12),

$$\begin{aligned} p_{z,\dot{z}}(z,\dot{z}) &= \int_0^\infty p_{z,\dot{z},y}(z,\dot{z},y)\,dy = \\ &= \int_0^\infty \frac{1}{y} p_{\dot{z}|yz}(\dot{z} \mid yz)\, p_y(y)\, p_x\left(\frac{z}{y}\right) dy. \end{aligned} \tag{13}$$

LCR of the product of Nakagami-m and kappa-mu random variables is:

$$\begin{aligned} N_Z &= \int_0^\infty |\dot{z}|\, p_{\dot{z},z}(\dot{z},z)\,d\dot{z} = \\ &= \int_0^\infty \frac{1}{y} p_y(y)\, p_x\left(\frac{z}{y}\right) dy \int_0^\infty \dot{z} p_{\dot{z}|yz}(\dot{z} \mid yz)\,d\dot{z} = \\ &= \int_0^\infty \frac{1}{y} p_y(y)\, p_x\left(\frac{z}{y}\right) \frac{1}{\sqrt{2\pi}} \sigma_{\dot{z}}\, dy, \end{aligned} \tag{14}$$

since,

$$\int_0^\infty \dot{z} \frac{1}{\sqrt{2\pi}\sigma_{\dot{z}}} e^{-\frac{\dot{z}^2}{2\sigma_{\dot{z}}^2}} d\dot{z} = \frac{1}{\sqrt{2\pi}} \sigma_{\dot{z}}. \quad (15)$$

After substitution (2) and (3) in (14) and by transforming (3) with [15, eq.03.02.06.0002.01] the LCR of the product of Nakagami-*m* and kappa-mu random processes can be obtained as:

$$N_z = \frac{\pi f_m}{\sqrt{2\pi}} \frac{2}{\Gamma(m_1)} \left(\frac{m_1}{\Omega_1}\right)^{m_1 - \frac{1}{2}} \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu} \Omega_2^{\frac{\mu+1}{2}}} \times \sum_{i=0}^{\infty} \left(\mu \sqrt{\frac{k(k+1)}{\Omega_1}}\right)^{2i+\mu-1} \frac{1}{i!\Gamma(i+\mu)} z^{(2m_1-1)}. \quad (16)$$

$$\int_0^\infty \sqrt{1 + \frac{m_1}{\Omega_1} \frac{\Omega_2}{\mu(k+1)} \frac{z^2}{y^4}} e^{-\frac{m_1}{\Omega_1}\frac{z^2}{y^2} - \frac{\mu(k+1)}{\Omega_2} y^2 + \ln y^{2i+2\mu-2m_1}} dy. \quad (17)$$

The previous integral is solved using Laplace approximation formula [8, 9]:

$$\int_0^\infty a(y) e^{-\lambda b(y)} dy \approx \sqrt{\frac{2\pi}{\lambda}} \frac{a(y_0)}{\sqrt{b''(y_0)}} e^{-\lambda b(y_0)}. \quad (18)$$

Expressions $a(y)$ and $b(y)$ are:

$$a(y) = \sqrt{1 + \frac{m_1}{\Omega_1} \frac{\Omega_2}{\mu(k+1)} \frac{z^2}{y^4}}, \quad (19)$$

$$b(y) = \frac{m_1}{\Omega_1} \frac{z^2}{y^4} + \frac{\mu(k+1)}{\Omega_2} y^2 - \ln y^{2i+2\mu-2m_1}. \quad (20)$$

The first and the second derivatives of $b(y)$ are, respectfully:

$$b'(y) = -2\frac{m_1}{\Omega_1}\frac{z^2}{y^3} + 2\frac{\mu(k+1)}{\Omega_2} y - (2i + 2\mu - 2m_1)\frac{1}{y}, \quad (21)$$

$$b''(y) = 6\frac{m_1}{\Omega_1}\frac{z^2}{y^4} + 2\frac{\mu(k+1)}{\Omega_2} + (2i + 2\mu - 2m_1)\frac{1}{y^2}. \quad (22)$$

where y_0 can be derived from the following expression:

$$b'(y_0) = 0, \quad (23)$$

$$y_0 = \left(\frac{(i+\mu-m_1) + \sqrt{(i+\mu-m_1)^2 - 4\frac{\mu(k+1)m_1}{\Omega_1\Omega_2} z^2}}{\frac{2\mu(k+1)}{\Omega_2}} \right)^{\frac{1}{2}}. \quad (24)$$

Finally, the LCR of the product of Nakagami-*m* and kappa-mu random processes as the infinite series expression is obtained:

$$N_z = \frac{\pi f_m}{\sqrt{2\pi}} \frac{2}{\Gamma(m_1)} \left(\frac{m_1}{\Omega_1}\right)^{m_1 - \frac{1}{2}} \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu} \Omega_2^{\frac{\mu+1}{2}}} \times \sum_{i=0}^{\infty} \left(\mu \sqrt{\frac{k(k+1)}{\Omega_1}}\right)^{2i+\mu-1} \frac{1}{i!\Gamma(i+\mu)} \times z^{(2m_1-1)} \sqrt{\frac{2\pi}{\lambda}} \frac{a(y_0)}{\sqrt{b''(y_0)}} e^{-\lambda f(y_0)}. \quad (25)$$

LCR OF THE PRODUCT OF THREE RANDOM PROCESSES

In this chapter, the product of three Nakagami-*m* random process is considered:

$$x = x_1 x_2 x_3, \quad (26)$$

where x_i, i=1, 2, 3 follows Nakagami-*m* distribution:

$$p_{x_i}(x_i) = \frac{2}{\Gamma(m_i)} \left(\frac{m_i}{\Omega_i}\right)^{m_i} x^{2m_i-1} e^{-\frac{m_i}{\Omega_i} x_1^2}, \quad i = 1,2,3. \quad (27)$$

where, as before m_i is the fading severity parameter and Ω_i is the mean value of Nakagami-*m* process. Similarly, as in previous chapter, the first derivative of x is:

$$\dot{x} = x_2 x_3 \dot{x}_1 + x_1 x_3 \dot{x}_2 + x_1 x_2 \dot{x}_3. \quad (28)$$

Taking into account that $\dot{x}_1$, $\dot{x}_2$ and $\dot{x}_3$ are Gaussian random processes, the variance of $\dot{x}$ is:

$$\sigma_{\dot{x}}^2 = x_2^2 x_3^2 \sigma_{\dot{x}_1}^2 + x_1^2 x_3^2 \sigma_{\dot{x}_2}^2 + x_1^2 x_2^2 \sigma_{\dot{x}_3}^2, \quad (29)$$

where,

$$\sigma_{\dot{x}_i}^2 = \pi^2 f_m^2 \frac{\Omega_i}{m_i}, \quad i = 1,2,3. \quad (30)$$

After substitution (30) in (29), expression for variance of $\dot{x}$ is:

$$\sigma_{\dot{x}}^2 = \frac{\pi^2 f_m^2 \Omega_1}{m_1} x_2^2 x_3^2 \left(1 + \frac{x^2}{x_2^4 x_3^2} \frac{\Omega_2 m_1}{\Omega_1 m_2} + \frac{x^2}{x_2^2 x_3^4} \frac{\Omega_3 m_1}{\Omega_1 m_3}\right). \quad (31)$$

JPDF of x, $\dot{x}$, x_2 and x_3 is:

$$p_{x,\dot{x},x_2,x_3}(x,\dot{x},x_2,x_3) = p_{\dot{x}}(\dot{x} \mid x x_2 x_3) p_{x x_2 x_3}(x x_2 x_3) = p_{\dot{x}}(\dot{x} \mid x x_2 x_3) p_{x_2}(x_2) p_{x_3}(x_3) p_x(x \mid x_2 x_3), \quad (32)$$

where,

$$p_z\left(x \mid x_2 x_3\right)=\left|\frac{dx_1}{dx}\right| p_{x_1}\left(\frac{x}{x_2 x_3}\right), \quad \frac{dx_1}{dx}=\frac{1}{x_2 x_3}. \tag{33}$$

JPDF of x and $\dot{x}$ can be expressed as:

$$p_{x,\dot{x}}\left(x,\dot{x}\right)=\int_0^\infty dx_2 \int_0^\infty p_{x,\dot{x},x_2,x_3}\left(x,\dot{x},x_2,x_3\right) dx_3. \tag{34}$$

Finally, LCR of the product of three Nakagami-*m* processes is:

$$\begin{aligned} N_x &= \int_0^\infty |\dot{x}|\, p_{\dot{x},x}\left(\dot{x},x\right) d\dot{x} = \frac{\pi f_m}{\sqrt{2\pi}}\left(\frac{m_1}{\Omega_1}\right)^{\frac{1}{2}}\left(\frac{m_1}{\Omega_1}\right)^{m_1} \\ &\times\left(\frac{m_2}{\Omega_2}\right)^{m_2}\left(\frac{m_3}{\Omega_3}\right)^{m_3}\frac{8}{\Gamma\left(m_1\right)\Gamma\left(m_2\right)\Gamma\left(m_3\right)} x^{2m_1-1} \\ &\times\int_0^\infty dx_2\int_0^\infty dx_3\left(1+\frac{x^2}{x_2^{\,4}x_3^{\,2}}\frac{\Omega_2 m_1}{\Omega_1 m_2}+\frac{x^2}{x_2^{\,2}x_3^{\,4}}\frac{\Omega_3 m_1}{\Omega_1 m_3}\right)^{\frac{1}{2}} \\ &\times e^{-\frac{m_1}{\Omega_1}\frac{x^2}{x_2^{\,2}x_3^{\,2}}-\frac{m_2}{\Omega_2}x_2^{\,2}-\frac{m_3}{\Omega_3}x_3^{\,2}+2(m_2-m_1)ln\, x_2+2(m_3-m_1)ln\, x_3}. \end{aligned} \tag{35}$$

The two-folded integral in (35) can be solved using Laplace approximation formula [8]:

$$\begin{aligned} &\int_0^\infty dx_2\int_0^\infty g\left(x_2,x_3\right)e^{-\lambda f\left(x_2,x_3\right)}dx_3 \\ &\approx \frac{2\pi}{\lambda}\frac{g\left(x_{2m},x_{3m}\right)}{\sqrt{det\beta}}e^{-\lambda f\left(x_{2m},x_{3m}\right)}, \end{aligned} \tag{36}$$

where,

$$\beta=\begin{vmatrix} \dfrac{\partial^2 f\left(x_{2m},x_{3m}\right)}{\partial x_{2m}^{\;2}} & \dfrac{\partial^2 f\left(x_{2m},x_{3m}\right)}{\partial x_{2m}\partial x_{3m}} \\ \dfrac{\partial^2 f\left(x_{2m},x_{3m}\right)}{\partial x_{3m}\partial x_{2m}} & \dfrac{\partial^2 f\left(x_{2m},x_{3m}\right)}{\partial x_{3m}^{\;2}} \end{vmatrix}, \tag{37}$$

$$\frac{\partial f\left(x_{2m},x_{3m}\right)}{\partial x_{2m}}=0, \tag{38}$$

$$\frac{\partial f\left(x_{2m},x_{3m}\right)}{\partial x_{3m}}=0. \tag{39}$$

LCR OF THE PRODUCT OF FOUR RANDOM PROCESSES

This chapter presents derivation of closed form expression for LCR of the product of four Nakagami-*m* processes using Laplace approximation formula for three-folded integrals. Similarly, the product of the four Nakagami-*m* random process is:

$$y=y_1 y_2 y_3 y_4, \tag{40}$$

where y_i, i=1, 2, 3, 4 follows Nakagami-*m* distribution:

$$p_{y_i}\left(y_i\right)=\frac{2}{\Gamma\left(m_i\right)}\left(\frac{m_i}{\Omega_i}\right)^{m_i} y^{2m_i-1}e^{-\frac{m_i}{\Omega_i}y_i^{\,2}}, i=1,2,3,4. \tag{41}$$

The first derivative of the product of four Nakagami-*m* random processes is:

$$\dot{y}=y_2 y_3 y_4 \dot{y}_1+y_1 y_3 y_4 \dot{y}_2+y_1 y_2 y_4 \dot{y}_3+y_1 y_2 y_3 \dot{y}_4. \tag{42}$$

Random process $\dot{y}$ has conditional Gaussian distribution, since $\dot{y}_1$, $\dot{y}_2$, $\dot{y}_3$ and $\dot{y}_4$ are Gaussian random processes. The variance of $\dot{y}$ is:

$$\begin{aligned} \sigma_{\dot{y}}^2 &= y_2^{\,2}y_3^{\,2}y_4^{\,2}\sigma_{\dot{y}_1}^2+y_1^{\,2}y_3^{\,2}y_4^{\,2}\sigma_{\dot{y}_2}^2 \\ &+y_1^{\,2}y_2^{\,2}y_4^{\,2}\sigma_{\dot{y}_3}^2+y_1^{\,2}y_2^{\,2}y_3^{\,2}\sigma_{\dot{y}_4}^2, \end{aligned} \tag{43}$$

where,

$$\sigma_{\dot{y}_i}^2=\pi^2 f_m^{\;2}\frac{\Omega_i}{m_i}, \quad i=1,2,3,4. \tag{44}$$

From previous expressions, variance of $\dot{y}$ becomes:

$$\begin{aligned} \sigma_{\dot{y}}^2 &= \frac{\pi^2 f_m^{\;2}\Omega_1}{m_1}y_2^{\,2}y_3^{\,2}y_4^{\,2}\left(1+\frac{y^2}{y_2^{\,4}y_3^{\,2}y_4^{\,2}}\frac{\Omega_2 m_1}{\Omega_1 m_2}\right. \\ &\left.+\frac{y^2}{y_2^{\,2}y_3^{\,4}y_4^{\,2}}\frac{\Omega_3 m_1}{\Omega_1 m_3}+\frac{y^2}{y_2^{\,2}y_3^{\,2}y_4^{\,4}}\frac{\Omega_4 m_1}{\Omega_1 m_4}\right). \end{aligned} \tag{45}$$

JPDF of y, $\dot{y}$, y_2, y_3 and y_4 is:

$$\begin{aligned} p_{y,\dot{y},y_2,y_3,y_4}\left(y,\dot{y},y_2,y_3,y_4\right) &= p_{\dot{y}}\left(\dot{y}\mid y y_2 y_3 y_4\right) \\ &\times p_{y_2}\left(y_2\right)p_{y_3}\left(y_3\right)p_{y_4}\left(y_4\right)p_y\left(y\mid y_2 y_3 y_4\right). \end{aligned} \tag{46}$$

JPDF of y and $\dot{y}$ can be expressed as:

$$p_{y,\dot{y}}\left(y,\dot{y}\right)=\int_0^\infty dy_2\int_0^\infty dy_3\int_0^\infty p_{y,\dot{y},y_2,y_3,y_4}\left(y,\dot{y},y_2,y_3,y_4\right)dy_4. \tag{47}$$

LCR for this case is:

$$\begin{aligned} N_y &= \int_0^\infty |\dot{y}|\, p_{\dot{y},y}\left(\dot{y},y\right)d\dot{y}=\frac{16}{\Gamma\left(m_1\right)\Gamma\left(m_2\right)\Gamma\left(m_3\right)\Gamma\left(m_4\right)} \\ &\times\frac{\pi f_m}{\sqrt{2\pi}}\left(\frac{m_1}{\Omega_1}\right)^{\frac{1}{2}}\left(\frac{m_1}{\Omega_1}\right)^{m_1}\left(\frac{m_2}{\Omega_2}\right)^{m_2}\left(\frac{m_3}{\Omega_3}\right)^{m_3}\left(\frac{m_4}{\Omega_4}\right)^{m_4} \\ &\times y^{2m_1-1}\int_0^\infty dy_2\int_0^\infty dy_3\int_0^\infty dy_4\left(1+\frac{y^2}{y_2^{\,4}y_3^{\,2}y_4^{\,2}}\frac{\Omega_2 m_1}{\Omega_1 m_2}+\frac{y^2}{y_2^{\,2}y_3^{\,4}y_4^{\,2}}\right. \\ &\left.\times\frac{\Omega_3 m_1}{\Omega_1 m_3}+\frac{y^2}{y_2^{\,2}y_3^{\,2}y_4^{\,4}}\frac{\Omega_4 m_1}{\Omega_1 m_3}\right)^{\frac{1}{2}}e^{-\frac{m_1}{\Omega_1}\frac{y^2}{y_2^{\,2}y_3^{\,2}}\frac{1}{y_4^{\,2}}-\frac{m_2}{\Omega_2}y_2^{\,2}-\frac{m_3}{\Omega_3}y_3^{\,2}} \\ &\times e^{-\frac{m_4}{\Omega_4}y_4^{\,2}\,2\left(m_2-m_1\right)ln\, y_2+2\left(m_3-m_1\right)ln\, y_3+2\left(m_2-m_1\right)ln\, y_4}. \end{aligned} \tag{48}$$

The closed form solution of three-folded integral LCR expression of the product of four Nakagami-*m* processes can be obtained by applying Laplace approximation formula [8], as follows:

$$\int_0^\infty dy_2 \int_0^\infty dy_3 \int_0^\infty g\left(y_2, y_3, y_4\right) e^{-\lambda f\left(y_2, y_3 y_4,\right)} dy_4 \approx \left(\frac{\pi}{\lambda}\right)^{\frac{3}{2}} \frac{g\left(y_{20}, y_{30}, y_{40}\right)}{\sqrt{det\beta}} e^{-\lambda f\left(y_{20}, y_{30} y_{40}\right)}. \tag{49}$$

where,

$$\beta = \begin{vmatrix} \frac{\partial^2 f\left(y_{20}, y_{30}, y_{40}\right)}{\partial y_{20}{}^2} & \frac{\partial^2 f\left(y_{20}, y_{30}, y_{40}\right)}{\partial y_{20}\partial y_{30}} & \frac{\partial^2 f\left(y_{20}, y_{30}, y_{40}\right)}{\partial y_{20}\partial y_{40}} \\ \frac{\partial^2 f\left(y_{20}, y_{30}, y_{40}\right)}{\partial y_{30}\partial y_{20}} & \frac{\partial^2 f\left(y_{20}, y_{30}, y_{40}\right)}{\partial y_{30}{}^2} & \frac{\partial^2 f\left(y_{20}, y_{30}, y_{40}\right)}{\partial y_{30}\partial y_{40}} \\ \frac{\partial^2 f\left(y_{20}, y_{30}, y_{40}\right)}{\partial y_{40}\partial y_{20}} & \frac{\partial^2 f\left(y_{20}, y_{30}, y_{40}\right)}{\partial y_{40}\partial y_{30}} & \frac{\partial^2 f\left(y_{20}, y_{30}, y_{40}\right)}{\partial y_{40}{}^2} \end{vmatrix}, \tag{50}$$

$$\frac{\partial f\left(y_{20}, y_{30}, y_{40}\right)}{\partial y_{20}} = 0, \tag{51}$$

$$\frac{\partial f\left(y_{20}, y_{30}, y_{40}\right)}{\partial y_{30}} = 0, \tag{52}$$

$$\frac{\partial f\left(y_{20}, y_{30}, y_{40}\right)}{\partial y_{40}} = 0. \tag{53}$$

NUMERICAL RESULTS

The normalized LCR versus *z* in dB for various values of Nakagami-*m* average power Ω_1, kappa-mu average power Ω_2, Nakagami-*m* fading severity parameter m_1, kappa-mu severity fading parameter μ and constant values of dominant factor k, is presented on Figure 1.

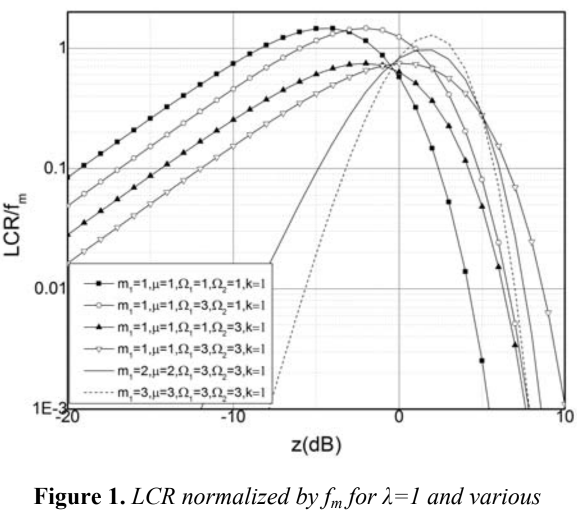

Figure 1. *LCR normalized by f_m for λ=1 and various parameters values m_1, μ, Ω_1, Ω_2 and constant values of k*

The LCR increases for small values of *z*, reaches its maximum and then decreases for higher values of *z*. When Nakagami-*m* envelope average power or kappa-mu average power increases, for lower values of *z*, LCR decreases, which leads to the improvement of the performances. It is obvious that the influence of kappa-mu envelope average power is greater than Nakagami-*m* envelope average power. Furthermore, LCR significantly decreases when fading severity parameter increases, as expected.

Figure 2 presents normalized LCR for various values of parameter k and constant values of Ω_1, Ω_2, m_1 and μ. It can be seen that for lower values of *z*, LCR increases as parameter k increases. On the contrary, for higher values of *z*, LCR decreases as parameter k increases.

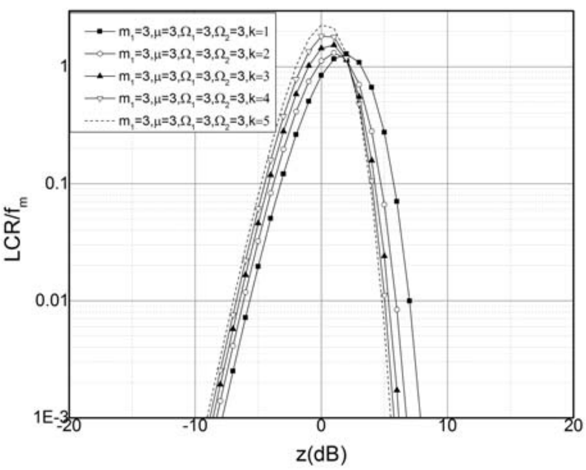

Figure 1. *LCR normalized by f_m for λ=1 and constant values of m_1, μ, Ω_1, Ω_2 and different values of k*

CONCLUSION

Laplace approximation formula has been used to derive the LCR of the product of Nakagami-*m* and kappa-mu random processes as infinite series expression. Obtained results can be used in performance analysis of wireless dual-hop relay communication system operating over multipath fading channel when one section is subjected to NLOS multipath fading environment while another section is subjected to LOS multipath fading environment. Further, LCR can be used to evaluate average fade duration of the product of Nakagami-*m* and kappa-mu random processes. Numerical results are graphically presented to show the influence of Nakagami-*m* envelope average power, kappa-mu envelope average power, Rician dominant component and fading severity parameter on LCR. System parameters modeling can lead to more reliable and efficient transmission which is of tremendous importance in risky environments. Moreover, two-folded and three folded integrals have been efficiently approximated by Laplace formula for derivation of closed form expression for LCR of the product of three and four Nakagami-*m* random processes.

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PRIMENA LAPLASOVE APROKSIMACIONE FORMULE U ANALIZI PERFORMASI BEŽIČNOG RELEJNOG KOMUNIKACIONOG SISTEMA U SREDINI SA MULTIPATH FEDINGOM

Časlav Stefanović

Rezime: *U ovom radu, Laplasova aproksimaciona formula uspešno je primenjena za izračunanje izraza za srednji broj osnih preseka proizvoda Nakagami-m i kappa-mu slučajnih procesa. Rezultat se može upotrebiti u analizi performansi relejnih sistema sa dve deonice u specifičnom multipath feding okruženju kada je prva deonica pod uticajem multipath feding sredine bez prisustva optičke vidljivosti a druga deonica je pod uticajem multipath feding sredine sa prisustvom optičke vidljivosti. Uticaji multipath feding parametra na srednji broj osnih preseka predloženog modela su ispitani, grafički predstavljeni i obrazloženi. Analitički pristup primene Laplasove aproksimacione formule u relejnim sistema sa više deonica je zatim razmatran prilikom izvodjenja izraza u zatvorenom obliku za srednji broj osnih preseka proizvoda tri i četri Nakagami-m procesa, redom.*

Ključne reči: srednji broj osnih preseka, kappa-mu multipath feding, laplasova aproksimaciona formula, Nakagami-m raspodela, relejni komunikacioni sistem.