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A NEW APPROACH FOR DETERMINING THE SYSTEM TIME WITHOUT FAILURES USING PETRI NETS

Abstract: *A new method for determining the time without failures using Petri nets will be presented in this paper. Petri nets are very appropriate for modelling and analysis of different systems. Due to its simplicity, this method can be applicable in practice, for example, for reliability analysis and for the time without failures calculation. The results are illustrated using the example of electric power system.*

Key words: Petri nets, time without failures, fault tree

INTRODUCTION

The scientific and technical progress of systems resulted in appearance and development of reliability theory. Complex systems should have high reliability considering the consequences of their failures influencing on further work and people security. Reliability $R(t)$ is usually defined as the probability of system work without failures in certain period of time and given environmental conditions. Reliability is defined by the following relation:

$$R(t) = \int_{t_1}^{\infty} p(t)dt \qquad (1)$$

where $p(t)$ is probability density distribution.

Since, the moments of failures and time without failures are random variables, reliability theory is based on the application of probability theory and mathematics statistic.

During the exploitation, system is under influence of different, exterior and interior, factors, i.e., wear, corrosion, aging etc. These factors can change the system characteristics and have strong influence on its work. Also, they can change the values of system parameters and its reliability and in the limit case can bring system to the instability or failure. Since, failures are random variables, system state is random, also.

One method for the reliability analysis is the application of Petri nets. Petri Nets are a graphical and mathematical modeling tool applicable to many systems, [1-7]. They are used for describing and studying information processing systems that are characterized as being concurrent, asynchronous, distributed, parallel, nondeterministic, and/or stochastic. As a graphical tool, Petri Nets can be used for a visual-communication similar to block diagrams. Tokens are used in these nets to simulate the dynamic of systems. As a mathematical tool, it is possible to set up the state equations, algebraic equations, and other mathematical models which describe behaviour of dynamical system.

Petri nets are very appropriate for reliability modelling and analysis of different systems. Petri nets can be applied in the field of systems safety, logistic, industry, computer science, etc. They are simple, easy to expand and analysed using simulation. Petri nets are the only class of graphs allowing complete analysis of reliability, [8-12].

If some of the system state coordinates are random variables, Stochastic Petri nets can be used for analysis of such systems.

Fault tree method is very applicable for reliability analysis and system safety. Using this method is easy to determine which combination of system elements can lead system to the failure. Based on fault tree is easy to obtain Petri net.

1. THE BASIC ELEMENTS OF PETRI NETS

Petri net is a special case of directed graph with initial state called initial marking, M_0. Also, Petri net represents bipartite, weighted graph consisting of two kinds of nodes called places and transitions. Nodes are connected by arcs. Arcs are directed either from place to transition or from transition to place. Places are denoted by circles and transitions by bars or boxes. Arcs are labeled with their weights (positive integers), so the *k* – weighted arc can be interpreted as a set of *k* parallel arcs. Unity weight is usually omitted. A place is an input place to a transition if there exists a directed arc connecting this place to the transition. A place is an output place of a transition if there exists a directed arc connecting the transition to the place.

For instance, input (output) places may represent preconditions (postconditions), and transition an event. Input places may represent the availability of resources, transition their utilization and output places the release of the resources.

Each place *p* is marked with nonnegative integer *k*. It is said that the place is marked with *k* tokens. Marking is denoted by *m* - vector *M*, where *m* is a total number of places. The *p*th element of vector *M*, denoted as *M(p)*, represents the numbers of tokens in place *p*.

The presence or absence of a token in a place can indicate whether a condition associated with this place is true or false, for instance. For a place representing the availability of resources, the number of tokens in this place indicates the number of available resources. At any given time instance, the distribution of tokens on places, called Petri Net marking, defines the current state of the modeled system.

In its simplest form, a Petri Net may be represented by a transition together with its input and output places. This elementary net may be used to represent various aspects of the modelled systems.

Petri Net can be defined as *PN=(P, T, I, O, Mo)*, where

1. $P=\{p_1,p_2,\dots,p_m\}$ is a finite set of places,
2. $T=\{t_1,t_2,\dots,t_n\}$ is a finite set of transitions, $P\cup T\neq\varnothing$, and $P\cap T=\varnothing$,
3. $I:(P\times T)\mapsto N$ is an input function defining directed arcs from places to transitions, where N is a set of nonnegative integers,
4. $O:(P\times T)\mapsto N$ is an output function defining directed arcs from transitions to places, and
5. $M_0 : P\mapsto N$ is the initial marking.

The following rules are used to govern the flow of tokens.

Enabling Rule: A transition *t* is said to be enabled if each input place *p* of *t* contains at least the number of tokens equal to the weight of the directed arc connecting *p* to *t*.

Firing Rule:

(a) An enabled transition *t* may or may not fire, and

(b) A firing of an enabled transition *t* removes from each input place *p* the number of tokens equal to the weight of the directed arc connecting *p* to *t*. It also adds in each output place *p* the number of tokens equal to the weight of the directed arc connecting *t* to *p*.

Classical Petri nets can be used only for the analysis of qualitative system characteristics because they do not contain the time concept. It is possible to describe only logical structure of the modelled system.

Introducing the time into Petri nets the Timed Petri nets are obtained. They can be used for the quantitative analysis of the system too. If time is random variable it is about Stochastic Petri nets (SPN).

In most Time Petri nets, time delay is given to the transitions. Only in some types of Petri nets, time delay is given to the places and/or arcs. Tokens of place p become unreachable to all input transitions for a certain time period. Petri nets like this is called *Timed Places Petri nets* (TPPNs).However, it is convinient to give delays to the transitions because they represents activities which perform in time. When transition becomes enabled it will fire after certain time. Petri nets like this is called *Timed Transitions Petri nets* (TTPNs).

2. SYSTEM MODELLING USING PETRI NETS

System with series connected elements represents the simpliest model for the analysis. In this system, the failure of one element causes the failure of the entire system.

The probability of the system work without failures can be increased decreasing the number of series connected elements or increasing the reliability of each element. It is obvious that the increase of the number of elements reduces the system reliability, [13, 14].

System with series connected elements can be modelled using Petri nets. In the next figure transitions are time transitions. However, it is possible to model this system using classical transitions, too.

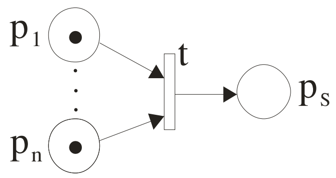

Figure 1. *Petri net of the system with series connected elements*

The presence of token in place p_i means that the appropriate element of the system, *i*, $i=1,\dots,n$, is reliable. Classical Petri nets is only possible to ascertain if the system is reliable or not (the presence or absence of token in the place p_S, respectively). Usually is necessary to determine with which probability the system is reliable. If time is assigned to the tokens, then the reliability of element depends on time for which token stays in a place. The system time without a failures, *T*, can be obtained on the next way:

$$T=\min\{t_{p_1},t_{p_2},\dots,t_{p_n}\} \tag{2}$$

where t_{p_i}, $i=1,\dots,n$ represents the time for which token stays in a place, i.e., the time interval in which the appropriate element works correctly. The time for which token stays in a place can be a random variable represented by some probability distribution. Each place has its own probability distribution of time for which token stays in a place.

System with parallel connected elements gives the possibility of the reliability increase in relation to the elements reliability. Parallel elements can be used as spare elements and when some element fails the system still works because the function of the failed element undertake other elements. The appropriate Stochastic Petri net for system with parallel connected elements is given in the next figure.

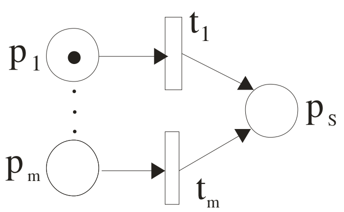

Figure 2. *Petri net of the system with parallel connected elements*

The system time without a failures, T, can be obtained on next way:

$$T = \max\{\tau_1, \ldots, \tau_m\} \qquad (3)$$

where $\tau_1, \ldots, \tau_m$ represents the time for which token stays in a place or time of transition firing.

In next figures composite systems are presented.

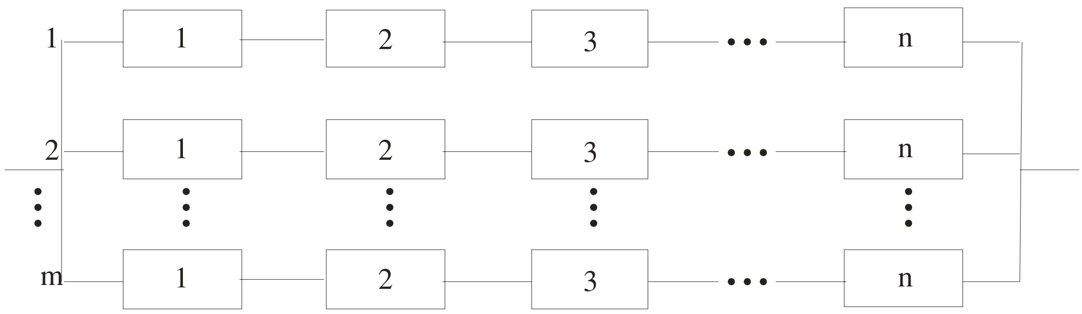

Figure 3. *Composite system*

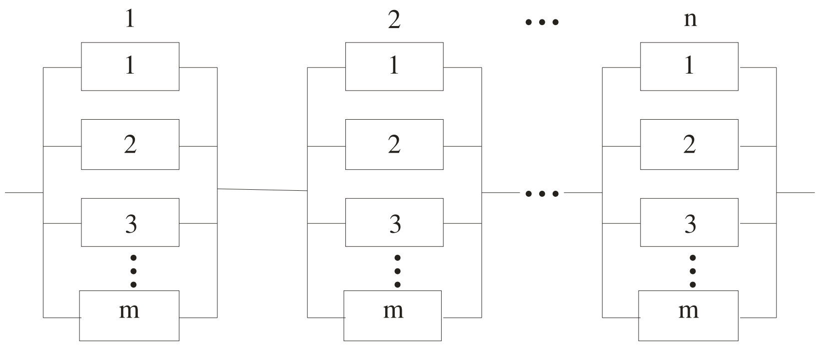

Figure 4. *Composite system*

In the next figures the appropriate Stochastic Petri nets are given.

The time without a failures, T, of the system can be obtained in the next way:

$$T = \max_i \min\{\tau_{i1}, \ldots, \tau_{in}\},\ i = 1, \ldots, m \qquad (4)$$

$$T = \min_i \max\{\tau_{i1}, \ldots, \tau_{im}\},\ i = 1, \ldots, n \qquad (5)$$

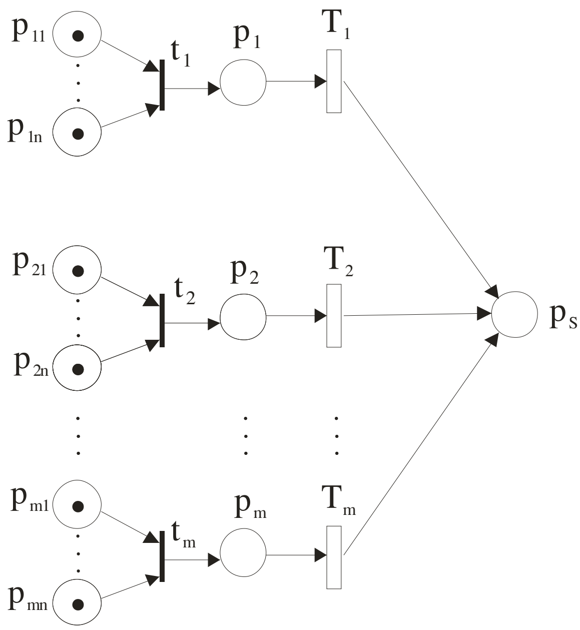

Figure 5. *Stochastic Petri net of Figure 3*

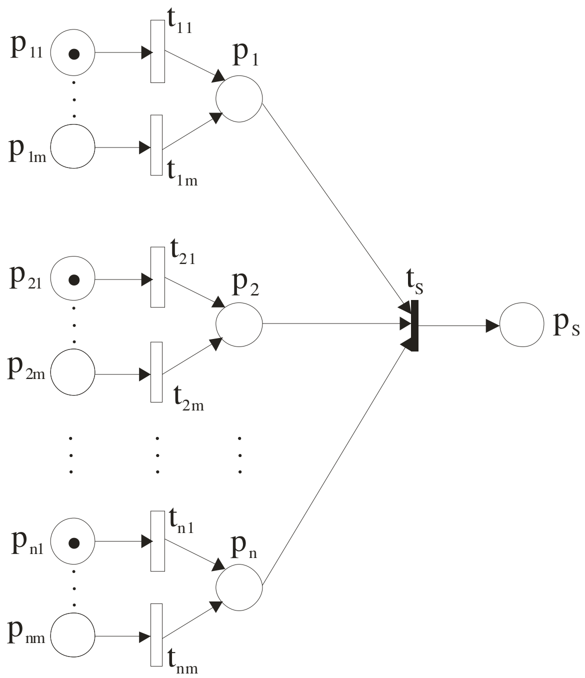

Figure 6. *Stochastic Petri net of Figure 4*

3. THE EXAMPLE OF TIME WITHOUT A FAILURES CALCULATION USING PETRI NETS

In this chapter a simple electric power system is used, [15-18]. From power plant *E* through the power lines L_1 and L_2 consumer *C* is powered. Consumer C is connected to the buses *S*. Buses *S* are supplied from the local source *G* which is connected over switch *B*.

For the reliability analysis of supplying consumer *C*, the next fault tree is used.

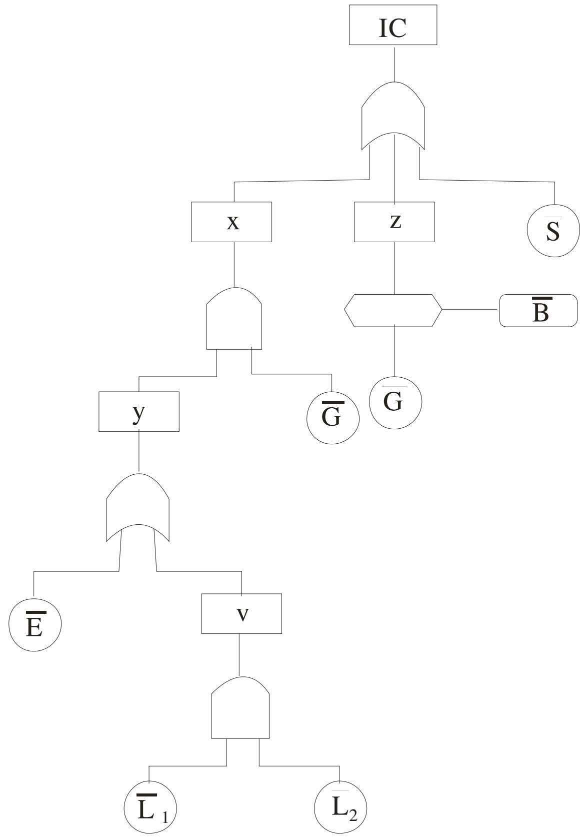

Figure 7. *The faul tree of electric power system*

Symbols in figure denote next events:

- *IC* - supply interruption of consumer *C*,
- *x* - malfunction at the same time of generator G and system for buses supply,
- *z* - malfunction of generator *G* and failure of switch *B*,
- *y* - malfunction at the same time of both wiring or malfunction of power plant *E*,
- *v* - malfunction at the same time of both wiring,
- $\overline{S}$ - malfunction of buses *S*,
- $\overline{E}$ - malfunction of power plant,
- $\overline{G}$ - malfunction of generator,
- $\overline{B}$ - failure of switch,
- $\overline{L}_1, \overline{L}_2$ - malfunction of both wirings L_1 and L_2.

Final event - failure of consumer *C* supply will appear when event *x* or *z* or $\overline{S}$ happen. Event *z* will happen when generator *G* is damaged and switch *B* can not turn off the generator because of the damage of switch itself or because of the failure of the protection that activates the switch. In that case, buses power must be stopped by turning off wirings L_1 and L_2. Event *x* will happened when both events *y* and $\overline{G}$ happen. Event *y* will appear when either $\overline{E}$ or *v* happen, and event *v* will appear when both wirings are in malfunction.

The appropriate Petri net is given in the next figure.

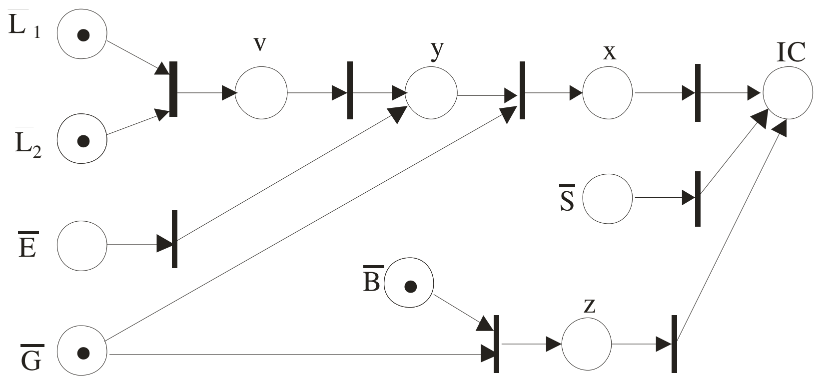

Figure 8. *Petri net of the electric power system*

The presence of tokens in a place denotes that certain event happened. Using classical Petri net it can be determined if the failure of consumer *C* appeared or not. Stochastic Petri nets can be used if we want to determine the probability of failure.

The system reliability is obtained using relation $R(t) = P\{T > t\}$, where T is the time without failures calculated in the fallowing way:

$$T = \max\{\tau_1, \tau_2, \tau_3\} \tag{6}$$

where:

τ_1 - is the time without failures of generator *G* and the system supplying buses,

τ_2 - is the time without failures of buses *S*,

τ_3 - is the time without failures of generator *G* and switch *B*,

$$\tau_1 = \min\{\tau_4, \tau_5\}$$

τ_4 - is the time without failures of both wirings or power plant *E*,

τ_5 - is the time without failures of generator *G*,

$$\tau_4 = \max\{\tau_6, \tau_7\}$$

τ_6 - is the time without failures of both wirings,

τ_7 - is the time without failures of power plant *E*,

$$\tau_6 = \min\{\tau_8, \tau_9\}$$

τ_8 - is the time without failures of wiring L_1,

τ_9 - is the time without failures of wiring L_2,

$$\tau_3 = \min\{\tau_5, \tau_{10}\}$$

τ_{10} - is the time without failures of switch *B*.

All of this time intevals refer to the time that token is in place or to the time necessary for transition to fire.

CONCLUSION

The appearance and development of reliability theory is a consequence of scientific and technical progress of the systems. For the reliability analysis Petri nets can be used successfully. Petri nets are a graphical and mathematical modeling tool applicable to many systems. Petri nets can be applied in the field of systems safety, logistic, industry, computer science, etc. They are simple, easy to expand and analysed using simulation. Petri nets are the only class of graphs allowing complete analysis of reliability. Fault tree method is very applicable for reliability analysis and system safety. Using this method is easy to determine which combination of system elements can lead system to the failure. Based on fault tree is easy to obtain Petri net.

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BIOGRAPHY

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NOVI PRISTUP ZA DOBIJANJE VREMENA BEZ OTKAZA SISTEMA KORIŠĆENJEM PETRI MREŽA

Bojana Zlatković, Biljana Samardzić

Apstrakt: *Novi način za određivanje vremena bez otkaza sistema korišćenjem Petri mreža je prikazan u ovom radu. Petri mreže su veoma pogodne za modeliranje i analizu različitih tipova sistema. Zbog svoje jednostavnosti, ovaj metod se može primeniti u praksi, na primer, za analizu pouzdanosti i izračunavanje vremena bez otkaza sistema. Rezultati su ilustrovani na primeru jednog elektroenergetskog sistema.*

Ključne reči: Petri mreže, vreme bez otkaza, stablo otkaza.