APPLICATION OF ESTIMATION METHOD FOR ANALYSING PILLARS’ GROUNDING SYSTEM FORMED OF SQUARE-SHAPED GROUND ELECTRODES

Abstract: The pillars’ grounding system formed of one and two horizontal square-shaped wire electrodes and an iron armature connected to the main grounding connector are analyzed in the paper. The procedure is based on recently proposed approach for modelling the influence of a concrete cylinder on a grounding system and application of estimation method for approximating a square-shaped electrode with a ring electrode.

Key words: ground inhomogeneity, grounding systems, estimation method, method of moments, quasi-stationary EM field.

INTRODUCTION

Pillars used as a part of the overhead power transmission, telecommunications, or lightning protection system necessarily include corresponding grounding systems. A basic electrode of such systems is usually formed of a basic star, rectangular or circularly shaped electrode [1-2] connected to an iron armature of the pillar foundation, which can be treated as a second part of the grounding system.

Official publications, as [1], usually neglect influence of concrete foundation. One simple procedure for approximate modelling of this influence, proposed in [3], is applied in this paper. It provides simplification of the analyzed problem to a problem of a grounding system in the homogeneous ground. This approach includes application of complex function for obtaining equivalent radius of the conductor’s cage. The level of foundation’s influence depends on electrical parameters of the concrete and the ground, which may have different values depending on the ground structure, humidity, etc.

Because of technical reasons in realization process, very often ground electrodes are square-shaped instead of being circularly-shaped. That is the reason why is the above mentioned procedure applied in this paper for the analysis of the grounding system that has the main electrodes’ system formed of one and two rectangular basic electrodes.

In this paper, the approach based on using estimation method is also used. It provides possibility for approximating grounding system formed from square-shaped with those consisting of circularly-shaped electrodes, since such problem is simpler from the numerical point of view.

Leakage currents’ distributions are assumed as constant while the earthing conductor’s influence is neglected. The surrounding ground is modeled as a homogeneous semi-conducting media of known electrical parameters, while the feeding current is modeled by an ideal low-frequency current generator. Calculations are carried out for various values of concrete’s specific resistivity and different embedding depths, whose values correspond to those of grounding systems realized in practice [1-2, 4-5].

THEORETICAL BACKGROUND

In order to provide easier reading of the paper content, a brief explanation of the procedure for modelling the system concrete foundation armature cage with single wire electrode of equivalent parameters [3,6], as well as the procedure for approximating the square-shaped with circularly-shaped wire-ground electrode will be given in the text which follows. Then, the description of the problem and corresponding procedure will be presented.

The procedure for modelling the foundation and wire armature cage as single wire electrode

The concrete pillar foundation is modelled as homogeneous parallelepipedical domain of specific conductivity $\sigma_0$, having a square cross-section with side length $b$, while the rest of the parameters can be noticed from Figure 1a. The surrounding ground is assumed as linear, isotropic and homogeneous media of specific conductivity $\sigma_1$. The armature placed inside foundation consists from $N$ vertical wire conductors of length $l_C$ having radii $r_0$ and the ends uniformly distributed at the circle of radii $a$ (upper part of Figure 1a). Applying the procedure based on using a complex function [6], a system of vertical armature electrodes is replaced by a single wire electrode having length $l_C$ and cross-section of radius $a_C = a \sqrt{N} r_0 / a$ placed in the concrete foundation.

Afterwards, applying the procedure, described in details in [5], the vertical electrode system and the concrete foundation can be replaced by a vertical electrode of equivalent length $l_1 = k_c l_C$ and cross-
section radius $a_e = K_e a_C$, placed in a homogeneous ground of specific conductivity $\sigma_1$, Figure 1b. Parameter $K_e$ is determined from the expression

$$K_e^{-1} = \frac{\sigma_1}{\sigma_C} \left(1 + \frac{l_e}{b} \right) \ln \frac{1 + \frac{l_e}{a_C}}{1 + \frac{l_e}{a_C}},$$

where $b = b_1 (1 + \sqrt{2}) / 4$, [3]. This way, the problem is reduced to analysis of the equivalent grounding system in the homogeneous ground shown in Figure 1b.

Figure 1 a) The vertical electrode system placed inside a concrete square cross-section foundation; b) An equivalent single vertical electrode placed in the homogeneous ground.

Approximation of wire square electrode with an equivalent ring electrode

As it has been already emphasized, the pillar grounding system realized with circular (instead of square) basic electrodes [7] is simpler for numerical solving. Because of that, it would be of interest to find some appropriate relationship between square side and radii of equivalent circular wire electrode that could approximately model the square electrode. A case of the square electrode being approximated by a circle of the same length and same surface, as well as the case of a circle having equivalent radii determined by estimation method [5], is analyzed in this paper. Related to the square side $c$, equivalent circle’s radii $R$ for those cases is

$$R = \frac{1}{2} \left( \frac{c \sqrt{2}}{2} + \frac{c}{2} \right) = \frac{c (1 + \sqrt{2})}{4}$$

(estimation method), (2a)

$$2R\pi = 4c \Rightarrow R = \frac{2c}{\pi}$$

(same length), and (2b)

$$R^2\pi = c^2 \Rightarrow R = \frac{c}{\sqrt{\pi}}$$

(same surface). (2c)

One square-shaped electrode grounding system

The pillar ground square electrode of side $c_1$, embedded at depth $h_1$ is observed, Figure 2. The electrode is made of a strip conductor (usually Fe-Zn strip) assumed to be a wire conductor having circular cross-section of an equivalent radius $a_1$ [8]. The geometry and parameters of the system armature-foundation have been already explained in chapter “The procedure for modelling the foundation and wire armature cage as single wire electrode”. Applying above mentioned procedure, the problem is reduced to analysis of the equivalent grounding system in homogeneous ground of specific conductivity $\sigma_1$ shown in Figure 3. By that is $l_1 = K_e l_C$ and $a_e = K_e a_C$, (3)

where $K_e$ is given in (1).

Figure 2 Pillar grounding system formed from one square-shaped electrode

Figure 3 Equivalent system of grounding system from Figure 2.
Equivalent vertical electrode (labelled by 1) and square wire electrode (labelled by 2) are fed by very low frequency currents $I_{g1}$ and $I_{g2}$, respectively. It is assumed that $a_k << l_1$, $a_1 << l_2 = 4c_1$ and $a_k, a_k << l_1$ ($l_1$ is wavelength in the surrounding ground, and $l_2 = 4c_1$ is length of the square wire electrode). Unknown longitudinal current distributions along vertical electrode (1) and square wire electrode (2) are labelled by $I_k(s'_k)$, $s'_k \in [0, l_k]$, $k = 1, 2$. Leakage currents’ distributions are assumed as constant,$ $2, 1 \lambda = \lambda_2, 1 = \lambda_1 \lambda$ (is wavelength in the surrounding ground, and $12, 4 \lambda = \lambda$ is length of the square wire electrode). Unknown longitudinal current distributions along vertical electrode (1) and square wire electrode (2) are labelled by $I_k(s'_k)$, $s'_k \in [0, l_k]$, $k = 1, 2$. Leakage currents’ distributions are assumed as constant,

$$I_{leak,k}(s'_k) = \frac{-\partial I_k(s'_k)}{\partial s'_k} = -I_k(s'_k) = \frac{I_{gk}}{l_k}, \ k = 1, 2. \quad (4)$$

Taking into consideration all presumptions pointed out in the previous text, as well as conditions:

$$I_1(0) = I_{g1}, \ I_2(0) = I_{g2} \text{ and } I_1(l_1) = 0, \quad (5)$$
electrical scalar potential at the point located in the vicinity of the grounding system shown in Figure 3 and defined by the field vector $\mathbf{r}$, can be determined using the expression:

$$\varphi(\mathbf{r}) = \sum_{k=1}^{2} \frac{l_{gk}}{4\pi \sigma_{l_k} l_{k}} \int \left( \frac{1}{r_k} + \frac{1}{r_{k1}} \right) d s'_{k}, \quad (6)$$

where $r_k$ and $r_{k1}$ denote distances between the current element, i.e. its image, and the field point, respectively. Applying the Method of Moments [9] and matching potential value on the $n$-th ($n = 1, 2$) conductor’s surface defined by the field vector $\mathbf{r}_1 = (l_1/2) \hat{z}$ (vertical conductor) and $\mathbf{r}_2 = 0.5c \hat{z}$ (square electrode), the following system of integral equations is formed:

$$\varphi(\mathbf{r} = \mathbf{r}_n) \equiv U_n, \ n = 1, 2, \quad (7)$$

In order to solve the described system, it is needed to adopt potential values $U_1$ and $U_2$. The system is solved in two regimes, so-called symmetric ($U_1 = U_2 = 1V$) and anti-symmetric ($U_1 = -U_2 = 1V$) feeding regimes. Based on these solutions, the solution for "$R$" parameters of the electrode system are obtained [7, 10]. They are formulated as

$$U_1 = R_{11} I_{g1} + R_{12} I_{g2}, \ U_2 = R_{21} I_{g1} + R_{22} I_{g2}, \quad (8)$$

where $R_{11}$ and $R_{22}$ are self-resistances, while $R_{12}$ and $R_{21}$ are mutual-resistances of the wire electrode (labelled with 1) and the square electrode (labelled with 2). "$R$" parameters represent integral grounding system characteristics and indicate level of mutual influence between two electrodes. For linear systems $R_{12} = R_{21}$. If electrodes are connected, i.e. they form a unique grounding system, substituting $U_1 = U_2 = 1V$ in (6), grounding impedance can be determined as

$$R_T = \frac{1}{I_{g1} + I_{g2}} = \frac{R_{11} R_{22} - R_{12} R_{21}}{R_{11} + R_{22} - R_{12} - R_{21}}, \quad (9)$$

The case of square electrode being approximated by a circle using one of expression (2a-c) is also analyzed in this paper. The mathematical model has the same form, and length $l_2$ in (6) in that case labels contour of ring electrode, instead of square wire electrode. Matching point is chosen at ring’s surface, and presumption of constant leakage current also stays valid.

![Figure 4](image_url)

**Figure 4** Pillar grounding system formed of two square-shaped electrodes.

![Figure 5](image_url)

**Figure 5** Equivalent system of grounding system from Figure 4.

### Two square-shaped electrodes grounding system

The pillar foundation grounding system formed of two square-shaped wire electrodes of sides $c_1$ and $c_2$, and embedded at depths $h_1$ and $h_2$, is observed (Figure 4). The electrodes are made of a strip conductor (usually Fe-Zn strip) assumed to be a wire conductor having circular cross-section of an equivalent radius $a_1$[8]. The geometry and parameters of the system armature-foundation have been the same as in the above presented problem of one square-electrode grounding system. Applying procedure for modelling foundation and wire armature cage as a single wire electrode, the problem is reduced to analysis of the equivalent
grounding system in homogeneous ground of specific conductivity $\sigma_1$ shown in Figure 5. Parameters of the wire conductor equivalent to the armature-foundation system are given with (3). This way, the problem is reduced to analysis of the equivalent grounding system in the homogeneous ground shown in Figure 5. Equivalent vertical electrode (labelled by 1) and square-shaped wire electrodes' system (electrodes labelled by 2 and 3) are fed by very low frequency currents $I_{g1}$ and $I_{g2}$, respectively. For practical values it is reasonable to assume that $a_6 << l_1$, $a_4 << l_2, l_3$ and $a_{1,4} << \lambda_1$ ($l_2 = 4c_1$ and $l_3 = 4c_2$ are length of electrodes 2 and 3, while $\lambda_1$ is wavelength in the surrounding ground). Unknown longitudinal current distributions along the vertical electrode (1) and square wire electrodes (2 and 3) are labelled by $I_k(s_k')$ $s_k' \in [0, l_k]$, $k = 1, 2, 3$. As in the case of the grounding system from Figure 3, the leakage currents' distributions are assumed as constant,

$$I_{\text{leak}}(s_k') = \frac{-\partial I_k(s_k')}{\partial s_k'} = -\frac{I_k}{l_k}, k = 1, 2, 3. \quad (10)$$

In previous expression, $I_k$, $k = 1, 2, 3$ are total currents which leak from k-th electrode respectively. Since the influence of earthing conductors has not been taken into consideration, and $I_l(l_1) = 0$, the following conditions are satisfied

$$I_{L1} = I_{g1}, I_{L2} + I_{L3} = I_{g2}. \quad (11)$$

Taking into consideration all presumptions pointed out in the previous text, electric scalar potential at the points defined by the field vector $\vec{r}$ in the vicinity of the equivalent grounding system shown in Figure 5, can be determined using the expression

$$\varphi(\vec{r}) = \frac{1}{2} \int_{k=1}^{3} \left( \frac{1}{l_k} + \frac{1}{l_k} \right) d(s_k'), \quad (12)$$

where $r_k$ and $r_k'$ denote distances between the current element, i.e. its image, and the field point, respectively. Applying the Method of Moments and matching potential value on the n-th ($n = 1, 2, 3$) conductor’s surface defined by the field vector $\vec{r} = \left( l_1 / 2 \right) \hat{z}$ (vertical conductor), $\vec{r}_2 = 0.5c_1 \hat{x}$ and $\vec{r}_2 = 0.5c_2 \hat{x}$ (points on the square-shaped electrodes), the following system of integral equations is formed:

$$\varphi(\vec{r} = \vec{r}_1) \equiv U_1, \varphi(\vec{r} = \vec{r}_2) \equiv U_2, k = 2, 3. \quad (13)$$

The rest of the procedure for obtaining "R" parameters and total resistance of the grounding system is identical to these one applied on grounding system from Figure 3.

Similar as in the case of the grounding system from Figure 3, a problem of the square electrodes being approximated by a circle using one of expressions (2a-c) is also analysed in this paper. The mathematical model has the same form, and in that case $l_2$ and $l_3$ in (12) label contours of the ring electrodes, instead of square wire electrodes.

**NUMERICAL RESULTS**

Based on the presented model, the corresponding program packages are developed and applied for approximate solving of the pillar foundation grounding system formed of a vertical conductor in concrete cylinder and one or two square electrodes. The geometry parameters are selected according to the values from [1-2, 4], while used concrete specific conductivity values are chosen from the ones from [5].

Values of the parameters from Figs. 2 and 3 (one square ground electrode) are $\rho_1 = 1/\sigma_1 = 100 \Omega \text{m}$, $a = 0.25 \text{ m}$, $b_1 = 0.4 \text{ m}$, $b_2 = 0.2 \text{ m}$, $l_c = 2 \text{ m}$, $c_1 = 1 \text{ m}$, $a_1 = 9.7 \text{ mm}$, $r_0 = 0.007 \text{ m}$ (radii of armature conductors cross-section), $N = 10$, $b = 0.483 \text{ m}$ and $a_c = 0.18 \text{ m}$.

Mutual resistance $R_{12}$ of the system from Figure 2 versus embedding depth $h_1$, having concrete specific resistivity $\rho_c = 1/\sigma_c$ taken as parameter, is shown in Figure 6. It is noticeable that the position of the square electrode related to vertical electrode's system is important for the way how specific resistivity concrete/ground ratio ($\rho_c / \rho_1$) influences the mutual resistance.

Total resistance $R_T$ of the system from Figure 2, versus embedding depth $h_1$ and concrete specific resistivity $\rho_c = 1/\sigma_c$ taken as parameter is shown in Figure 7. Decreasing of total resistance with respect to increasing embedding depth is expected, as well as disposition of the graphs which correspond to different values of specific resistivity of concrete.

Mutual resistance $R_{12}$ of the system from Figure 2, versus the embedding depth $h_1$ when $\rho_1 = \rho_c$, for the square electrode and different radii of equivalent ring electrode (2a-c) is presented in Figure 8. It is obvious that best approximation of the square electrode is using a ring electrode of radii (2c) and (2a), depending on the part of the graphic, i.e. the embedding depth $h_1$.

In Figure 9, graphs of the self-resistance $R_{22}$ versus the embedding depth $h_1$ for the square electrode and different radii of equivalent ring electrode (2a-c) are presented. In this case, a ring having radii given by the expression (2a) best approximates the square electrode.

Finally, in Figure 10, the total resistance $R_T$ of the grounding system from Figure 2 versus embedding depth $h_1$ for square electrode and different radii of the equivalent ring electrode (2a-c) is shown. Similarly, as in the case of the mutual resistance $R_{12}$ (Figure 8), the best approximation of the square electrode is by a ring.
electrode of radii \((8c)\) and \((8a)\), depending on the embedding depth \(h_1\).

Used values for all parameters in Figs. 4 and 5 (two square-shaped ground electrodes) are:

- \(\rho_1 = 1/\sigma_1 = 100\,\Omega\,m\), \(a = 0.25\,m\), \(b_s = 0.4\,m\), \(b_1 = 0.5\,m\), \(a_1 = 9.7\,mm\), \(\rho_0 = 0.007\,m\) (radius of armature conductors’ cross-section), \(N = 10\), \(b = 0.483\,m\), and \(a_c = 0.18\,m\).

Mutual-resistance \(R_{12}\) of the system from Figure 4 versus embedding depth of the lower square electrode \(h_1\), having concrete specific resistivity \(\rho_c = \rho_1 = 1/\sigma_1\) taken as parameter, is shown in Figure 11. It is noticeable that the position of the lower square electrode related to vertical electrode's system influences the mutual resistance, and this influence differs for different specific resistivity concrete/ground ratio \((\rho_c/\rho_1)\).

**Figure 6.** Mutual resistance of the grounding system shown in Figure 2 versus embedding depth \(h_1\), when \(\rho_c\) is taken as a parameter.

**Figure 7.** Total resistance of the grounding system shown in Figure 2 versus embedding depth \(h_1\), when \(\rho_c\) is taken as a parameter.

**Figure 8.** Mutual-resistance of the grounding system shown in Figure 2 versus embedding depth \(h_1\), for \(\rho_1 = \rho_c\) and different dimensions of the equivalent ring electrode.

**Figure 9.** Self-resistance \(R_{22}\) corresponding to the basic electrode of the grounding system shown in Figure 2 versus embedding depth \(h_1\), for \(\rho_1 = \rho_c\) and different dimensions of the equivalent ring electrode.

**Figure 10.** Total resistance of the grounding system shown in Figure 2 versus embedding depth \(h_1\), for \(\rho_1 = \rho_c\) and different dimensions of the equivalent ring electrode.
Total resistance $R_T$ of the system from Figure 4, versus embedding depth $h_2$, and concrete specific resistivity $\rho_c = 1/\sigma_c$ taken as parameter, is shown in Figure 12. It is expected that the total resistance will decrease while increasing depth $h_2$, and also, that the curves corresponding to different values of specific resistivity of concrete are mutually shifted.

Mutual resistance $R_{12}$ of the system from Figure 4, versus the embedding depth $h_2$ when $\rho_1 = \rho_c$, for the square electrodes and different radii of equivalent ring electrodes (2a-c) is presented in Figure 13. It is obvious that best approximations of the square electrodes are the ring electrodes of radii (2c).

In Figure 14, graphs of the self-resistance $R_{22}$ versus the embedding depth $h_2$ when $\rho_1 = \rho_c$, for the square electrodes and different radii of equivalent ring electrodes (2a-c), are presented. In this case, rings having radii given by the expression (2a) are the best approximation of the square electrodes.

Finally, in Figure 15, the total resistance $R_T$ of the grounding system from Figure 4 versus the embedding depth $h_2$ when $\rho_1 = \rho_c$, for square electrodes and different radii of the equivalent ring electrode (2a-c) is shown. The best approximation of the square electrode is using ring electrodes of radii (2c) and (2a), depending on the embedding depth $h_2$. 

![Figure 11](image1) **Figure 11** Mutual resistance of the grounding system shown in Figure 4 versus embedding depth $h_2$, when $\rho_c$ is taken as a parameter.

![Figure 12](image2) **Figure 12** Total resistance of the grounding system shown in Figure 4 versus embedding depth $h_2$, when $\rho_c$ is taken as a parameter.

![Figure 13](image3) **Figure 13** Mutual-resistance of the grounding system shown in Figure 4 versus embedding depth $h_2$ for $\rho_1 = \rho_c$ and different dimensions of the equivalent ring electrodes.

![Figure 14](image4) **Figure 14** Self-resistance $R_{22}$ corresponding to the basic electrode of the grounding system shown in Figure 4 versus embedding depth $h_2$ for $\rho_1 = \rho_c$ and different dimensions of the equivalent ring electrodes.

![Figure 15](image5) **Figure 15** Total resistance $R_T$ of the grounding system from Figure 4 versus the embedding depth $h_2$ when $\rho_1 = \rho_c$, for square electrodes and different radii of the equivalent ring electrodes.
CONCLUSIONS

Based on the obtained results one can conclude that the influence of foundation on grounding system’s impedance can be significant for real values of concrete’s specific resistivity and it should be taken into consideration during the design of such systems. Since, from the numerical point of view, it is easier to solve an analogue grounding system with a ring basic electrodes’ system, different approaches for approximating a square electrode with an equivalent ring electrode are also presented in the paper.

Presented results indicate a possibility to successfully approximate the square electrode using an equivalent ring electrode, but the most appropriate approximations differ depending on the embedding depth of the basic electrode. Another collateral conclusion that follows the presented analysis is the significance of the influence of the shape of the electrodes, which form basic electrodes’ system, on the self-, mutual- and consequently, the total impedance. The obtained results show that selection of the approximating method given by Eqs. (2) depends on the number of the electrodes in the basic electrodes’ system, since the results for one square-shaped electrode grounding system differ a little bit from those ones for two-square-electrode grounding system.

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BIOGRAPHY

Nenad Cvetković was born in Niš, Serbia in 1970. He received the Dipl. Ing, MSc and PhD degrees in electrical engineering and MSc degrees from the Faculty of Electronic Engineering, University of Niš, Serbia, in 1995, 2002 and 2009, respectively. From 1995, he has been engaged as an associate on the Department of Theoretical Electrical Engineering, Faculty of Electronic Engineering of Niš. As an author or co-author, he has published about seventy papers in international journals or conference proceedings, one monograph dealing with grounding systems and two textbooks. His main research interests are numerical methods for electromagnetic field calculation, especially in transmission line and grounding systems analysing. Nenad Cvetković is currently working as an assistant professor at the Faculty of Electronic Engineering, University of Niš.
Rezime: U radu su analizirani uzemljivački sistemi stuba formirani od jedne, odnosno dve uzemljivačke elektrode i armature stuba priključene u uzemljivački sistem. Postupak je baziran na jednoj nedavno predloženoj proceduri za modelovanje uticaja betonskog temelja stuba na uzemljivački sistem i primeni metoda procene za aproksimaciju kvadratne konturne uzemljivačke elektrode, elektrodom kružnog oblika.

Ključne reči: potencijalna doza, primljena doza, interna doza, biološki efektivna doza, ekspozicija, zdravstveni rizik.